CURRICULUM VITAE

Dr. KANCHAN MISHRA
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Career Objective

To get a creative and challenging position of lecturer in which I can share my knowledge and utilize my experience to make the bright carrier of student as well as to gain more experience into the education and teaching field.

Educational Qualifications

Board/University	Examination	Academic Year	Percentage	Name of School/College
Rabindranath Tagore University (Bhopal)	PhD	2021	--	Rabindranath Tagore University(Bhopal)
The Global Open University (Nagaland)	M.Phil.	2009	70%	The Global Open University (Nagaland)
Pt.RSU(Raipur)	PGDCA	2008	62%	Maharaja Agrasen International College (Raipur)
Barkatullah University, Bhopal (M.P.)	M.Sc.(Maths)	2005	66%	NMV College (Narmadapuram)
Barkatullah University, Bhopal (M.P.)	B.Sc.(Maths)	2002	56%	Home Scienc College (Narmadapuram)
M.P.Board	Higher Secondary	1999	61%	Girls School (Narmadapuram)
M.P.Board	High School	1997	65%	Girls School (Narmadapuram)

Professional Experience

- Selected as a **guide in Department of Mathematics for PhD Scholar(Pt. Ravishankar Shukla University)**
- Working as Assistant Professor (Mathematics) in Vipra College, Raipur from 19/07/2019 Till date under section 28..
- Working as Assistant Professor (Mathematics) in Disha College, Raipur from 2014-2018 .
- Worked as a Asst. Professor (Mathematics) Maharaja Agrasen International College (MAIC), Samta Colony Raipur (C.G.) From 16.08.2010 to 30.05.2011
- Worked as a Teacher (Maths) Sadhu Vasvani Vidhya Mandri Higher Secondary School, Hoshangabad (M.P.) From 01.07.2005 to 30-11-2006.
- Worked as a Teacher (Maths) Maharshi Vidya Mandir Higher Secondary School, Hoshangabad (M.P.) From 20.11.2003 to 31.03.2004.

Key Responsibilities Handled

- Member of Research Cell Committee.
- Convener of IT Cell.
- Convener of Magazine Cell.
- Meet with parents to discuss progress and address behavior or mastery issues.
- Monitoring Attendance.
- Preparation of Question papers for the Mid-terms and Finals.
- Prepare lesson plans and methods of assessment.
- Making of Progress reports.
- Meeting with students and helping them in solving their problems.
- Extra-classes during College hours for the children willing to learn.
- Assist in admission and counseling.

Research Papers

1. International Journal of Advance Technology & Engineering Research (IJATER) , **(2017)**
   Paper Topic- 'Study of fixed point and common fixed point with Quasi metric space' UGC Approved (Paper Published).
2. International Journal of scientific progress and Research (IJSPR), **(2017)**
   Paper Topic- 'Study of fixed point in complete Metric Space by Symmetric Rational Expression' UGC Approved (Paper Published)
3. Mathematical Theory and Modling (IISTE) ,ISSN No 2224-5804(paper),ISSN 2225-0522(online),Vol 8 (2018).
   Paper Topic- Study of Soft set and Common Soft Point With Complete Metric Space. (Paper Published)
4. International Journal of Engineering, Applied and Management Sciences Paradigms (IJEAM),Issue 2,Vol. 54 , **UGC Approved** (2019).
   Paper Topic -Study of Fixed Point Theorems with b-Metric Space(Paper Published)
5. International Journal Of Scientific & Technology Research, Vol. 8,Issue 10, **UGC Approved** (2019).
   Paper Topic- Implementation of soft fixed point theorems with b-metric space(Paper Published)
6. International journal of Science and Research(IJSR),ISBN: 2319-7064(2021).
   Paper Topic -Implementation of Coupled Fixed Point Theorems with β- Monotone Property in Soft b-Metric space, (2022)UGC Approved (Paper Published)
7. Journal of Research and Development A Multidisciplinary International Level Referred Journal, Vol. -15,Issue -11, ISSN 2230-9578**.**
   Paper Topic -Implementation of Common Soft Fixed Point with Intuitionastic Soft Fuzzy Metric Space ,Impact Factor 7.265, UGC Approved.(2023)(Paper Published)
8. Journal of Network Security,STM JOURNALS, **"Advances in Data Security in Cryptography"**ISSN:2395-6739(Online),ISSN: 2321-8517(Print),Vol. 12,Issue 1,(2024)

9. **CompSci Journal Double Blind Peer Reviewed Open Access International Research Journal Paper topic-Impact of Common Fixed Point Theorems Involving Cubic Terms of d(p, q) in b-Metric Space ,ISSN: 3067-3089, Vol.-1,Issue-6,Dec.2025 ,Website:** *Website: https://csjour.com* |

Paper Presenting

- **International Conference on Resent Advances in Science, Technology and Management(ICRASTM-2025).Topic- Impact of Common Fixed Point Theorems involving Cubic Terms Of d(p,q) in b-Metric Spaces,(2025).**
- **International Innovation Horizon(IIH-2026) International Conference on Computational Intelligence, Quantum Computing and Emerging Science,(May 2026).**
- **International Conference on Resent Trends in Advance Science and Computing (ICRTASC-25).Topic- Application of Cubic Terms of d(p,q) in Soft b-metric spaces to Soft Common fixed Point. VIT BHOPAL ,(Nov 2025).**

Seminar Attened

1. International Workshop" FAIRNESS AND TRUST IN THE MATHEMATICAL MODELING AND MACHINE LEARNING TECHNIQUES" VIT BHOPAL(OCT 2023).
2. EMERGING AREAS IN COMPUTER SCIENCE AND TECHONOLOGY,VIPRA COLLEGE ,RAIPUR(CG)(2023).
3. One day national webinar on" Biodiversity, Ecosystem Services and Sustainabie Development, Raipur(2023)International seminar on " Indian Strategic interest in the Indian Ocean" Raipur(2023).
4. One day national webinar on" Role of Mathematics in real Life"Raigarh(MP)(2023).Seven Days Faculty development program " Teaching Pedagogy in Perspective of New Education Policy" Mahant Lakshaminarayan College Raipur(CG),(2023).
5. One week National workshop on " INTELLECTUAL PROPERTY RIGHTS(IPR) ,(2018).
6. International Conference on "Recent Advancement in Science & Technology" Bhopal(2017).
7. Faculty Development Program" Research Methodology for Innovative Research in Engineering and Applied Science" Bhopal(2018).

ACHIVMENTS

1. UGC –**Sponsored Refresher Course MMTC Program** Mathematics and Its Application ,Orgniesed by Silchar, Asam University **,Dec.2025. Grad A+.**
2. UGC –**Sponsored Refresher Course** 18th Sep. To 1st October 2024 Grade **A+.**
3. UGC-Sponsored **FIP "** Guru Dakshata" FACULTY INDUCTION PROGRAM 20/11/2023 To 19/12/2023 in GURU GASIDAS UNIVERSITY.
4. NEP 2020 **ORIENTATION & SENSTIZATION** PROGRAM ON 04/03/2024 To 13/03/2024 Obtained Grade A.

Hobbies

- Interaction with children
- Teaching
- Reading Books
- Listening Music
- Watching movies

Personal Information

Father's Name : Late. Mr.Ramyash Dwivedi
Mother's Name : Mrs. Geeta Dwivedi
Date of Birth : 7th April, 1981
Postal Address : 28, Central Avenue (West) Choubey Colony ,Raipur, Chhattisgarh.

Sex/Marital Status : Female / Married
Nationality : Indian
Languages : English, Hindi.
Educational Status : Ph. D (Mathematics)

Declaration & Signature

I hereby declare that above-mentioned information is correct up to my knowledge and I bear the responsibility for the correctness of the above-mentioned particulars.

Signature
……

पं. रविशंकर शुक्ल विश्वविद्यालय, रायपुर (छ.ग.)

दूरभाष : 0771-2262802 (अका.) 0771-2262540 (कुलसचिव) फैक्स -0771-2262811,2262807

क्रमांक- 3712 /अका./शोध/2026 रायपुर, दिनांक 06. 07. 2026

– आदेश –

गणित शोध उपाधि समिति की बैठक दिनांक 27.06.2026 द्वारा संशोधित अध्यादेश क्रमांक–45 की कंडिका–6 तथा रेगुलेशन–224 के अन्तर्गत लिये गए निर्णयानुसार निम्नलिखित आवेदकों को उनके नाम के सम्मुख दिये गए विषय में शोध–निर्देशक की मान्यता प्रदान की जाती है :–

क्र.	नाम/पता	विषय	संकाय
01	Dr. Kanchan Mishra Asst. Professor, Vipra Kala, Vanijya Avam Sharirik Shiksha Mahavidyalaya, Raipur	Mathematics	Science
02	Dr. Sumit Pant Asst. Professor, Asst. Professor, G. N.A. PG College, Bhatapara	Mathematics	Science
03	Dr. Anjali Chandravanshi Asst. Professor, Govr. J. Y. Chhattisgarh College, Raipur	Mathematics	Science
04	Dr. Jayesh Vijayrao Karanjgaonkar Asst. Professor, Swami Atmanand Govt. Englism Medium Adarsh College, Atari, Raipur	Mathematics	Science

आदेशानुसार,

उपकुलसचिव (अका.)

पृ. क्रमांक- 3713 /अका./शोध/2026 रायपुर, दिनांक 06 .07. 2026

प्रतिलिपि –

1. उपरोक्तानुसार शोध निदेशकों को सूचनार्थ अग्रेषित।
2. प्राचार्य, संबंधित महाविद्यालयों को सूचनार्थ अग्रेषित।
3. सहायक कुलसचिव (गोप.)/उपकुलसचिव (परीक्षा), पं. रविशंकर शुक्ल विश्वविद्यालय, रायपुर
4. कुलपति के सचिव/कुलसचिव के निजी सहायक, पं. रविशंकर शुक्ल विश्वविद्यालय, रायपुर को सूचनार्थ अग्रेषित।

उपकुलसचिव (अका.)

Goswami 06.7.26

पं. रविशंकर शुक्ल विश्वविद्यालय, रायपुर (छ.ग.)

दूरभाष : 0771–2262802 (अकादमिक), 0771–2262540 (कुलसचिव), फैक्स–0771–2262818, 2262607

क्र. 1764 /अका./अनुमोदन/2023 रायपुर,दिनांक 25 /04/2023

प्रति,

संचालक/प्राचार्य,
विप्र कला वाणिज्य एवं शारीरिक शिक्षा महाविद्यालय,
जी.ई.रोड, रायपुर,
रायपुर (छ.ग.)

विषयः–परिनियम 28 के तहत् शैक्षणिक पद पर नियुक्ति हेतु चयन समिति के अनुशंसा का अनुमोदन।

विषयांतर्गत आपके महाविद्यालय में विश्वविद्यालय परिनियम 28 के तहत् नियुक्ति हेतु विश्वविद्यालय द्वारा गठित चयन समिति द्वारा दिनांक 26.11.2022 को सहायक प्राध्यापक–वाणिज्य, दिनांक 24.12.2022 को सहायक प्राध्यापक–गणित, दिनांक 14.01.2023 को सहायक प्राध्यापक–कम्प्यूटर साइंस, दिनांक 19.01.2023 को सहायक प्राध्यापक–भौतिक, दिनांक 21.01.2023 को सहायक प्राध्यापक–शिक्षा एवं योगा के पद हेतु आयोजित साक्षात्कार में अनुशंसित उम्मीदवारों का निम्नानुसार अनुमोदन किया जाता हैः–

क	पदनाम	चयनित उम्मीदवार का नाम	वेतनमान
1	सहायक प्राध्यापक–वाणिज्य	1 सुश्री निधी श्री शुक्ला 2 डॉ.सीमा अग्रवाल	यू.जी.सी. द्वारा देय सातवॉ वेतनमान
2	Assistant Professor-Mathematics	1 Dr.Kanchan Mishra	
3	Assistant Professor-Computer Science	1 Ananya Pande Sharma 2 Dr.Raksha Badgaiyan 3 Dr.Deepshikha Sharma	
4	Assistant Professor – Education	1 Dr.Manisha Jain	
5	Assistant Professor –Physics	1 Devendra Kumar Golhani	
6	Assistant Professor –Yoga	1 Shri Lomesh Kumar 2 Shri Bhanu Prakash(W.L.) 3 Shri Ashish Kumar(W.L.)	

उपरोक्त चयनित उम्मीदवारों की नियुक्ति, पत्र निर्गमन की तिथि से मान्य होगी। चयनित उम्मीदवारों की नियुक्ति दो वर्ष की परीविक्षा अवधि हेतु अनुमोदित की गई है। संबंधित उम्मीदवारों को नियुक्ति आदेश रजिस्टर्ड डाक से 15 दिवस के भीतर भेजें तथा प्रतिलिपि अधोहस्ताक्षर कर्ता को भेजे एवं उनकी उपस्थिति प्रतिवेदन की प्रति के साथ विश्वविद्यालय को अवगत करायें। चयनित उम्मीदवारों को वेतन भुगतान बैंक के माध्यम से किया जाए।

परिनियम 28(31)(3) के अनुसार विश्वविद्यालय कार्यपरिषद के अनुमोदन के बिना किसी भी नियुक्त शिक्षकों को सेवा से पदच्युत नहीं किया जा सकेगा।

आदेशानुसार,

उप कुलसचिव (अका.)

पृ. क्रमांकः 1765/अका./अनुमोदन/2023 रायपुर, दिनाक :25/04/2023

प्रतिलिपि :

01. कुलपति के सचिव/कुलसचिव के निजी सहायक, पं. रविशंकर शुक्ल विश्वविद्यालय, रायपुर को सूचनार्थ।

कक्ष अधिकारी (अका.)

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Certificate of Publication

IJSPR/ECERT/V35/N3/1921

This to certify that paper titled "Study of Fixed Points in Complete Metric Space by Symmetric Rational Expression", authored by Mr. / Ms. Kanchan Mishra has published in International Journal of Scientific Progress and Research IJSPR, in Volume 35, Number 03, May 2017. The Paper has been passed through peer review process in terms of originality and quality of work by Reviewer Panel of IJSPR. The Paper is available online on following link:- http://www.ijspr.com/citations/v35n3/IJSPR_3503_1921.pdf

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International Journal of Scientific Progress and Research IJSPR

Organizer, IJSPR
International Journal of Scientific Progress & Research

Study of Fixed Points in Complete Metric Space by Symmetric Rational Expression

Kanchan Mishra[*a], Chitra Singh[*]

Department of Mathematics, AISECT University Bhopal, MP

Abstract: In this paper we introduce the different method to solve and prove some fixed point and common fixed pointtheorems for banach contraction in complete metric space by using new type rational contraction conditions. Our results extend and improve the recent ones.

Keywords: Fixed point, Common Fixed point, Continuous mapping, Complete Metric Space,cauchy sequence. AMS Classification: 45H10.

I. INTRODUCTION AND PRELIMINARIES:

Impact of fixed point theory in different branches of mathematics and its applications is immense. The first result on fixed points for contractive type mapping was the much celebrated Banach's contraction principle by S. Banach [1] in (1922). In the general setting of complete metric space, The important part of fixed point theory is Metric fixed point theory, because of its applications in different areas like variation and linear inequalities, improvement and approximation theory.Kannan[2]established fixed theorems in different spaces like Metric space. Rhoades (2001) and Khan et.al.(1984) author have proved that some unique fixed point theorems. In (1980) Jaggi[5] and Das [6] obtained some fixed point theorems with the mapping satisfying. Before going to prove our results first we define some important definitions.

Definition 2.1: Let (X, d) be a complete metric space, c ∈(0,1) and f: X → X be a mapping such that for each x,y∈X, $d(T_x, T_y) \leq$ cd (x, y) [1], Then f has a unique fixed point, a ∈ X,such that for each x ∈ X ,$\lim_{n\to\infty} f_{n_x} = a$. after the classical result, Kannan [2] gave a subsequently new contractive mapping to prove the fixed point theorem. Since then a number of mathematicians have been worked on fixed point theory dealing with mappings satisfying various type of contractive conditions. In (2002)A. Branciari [3] analyzed the existence of fixed point.

Definition 2.2: If T is self mapping of a complete metric space X into itself satisfying; $d(T_x, T_y) \leq \alpha$ [$d(T_x$, x)+ $d(T_y$,x)] for all x,y $\epsilon X$and $0 \leq \alpha \leq 1$,then T has unique fixed point in X.See[2]

Definition 2.3:$d(T_x, T_y) \leq$ [d(x, T_y)+ d(y, T_x)] for allx,y ϵX and $0 \leq \alpha \leq 1$,then T has unique fixed point in X.See[4], A similar conclusion was also obtained by Chaterjee [7].

Definition 2.4: Let the rational expression

$$d(T_x, T_y) \leq \alpha \frac{d(x, T_x)\, d(y, T_y)}{d(x,y)} + \beta\, d(x, y)$$

for allx ,y ϵX, x ≠y and $0 \leq \alpha + \beta \leq 1$

thenThas unique fixed point in X. See[5]

Definition 2.5: let some fixed point theorems with the mapping satisfying:

$$d(T_x, T_y) \leq \alpha \frac{d(x, T_x)\, d(y, T_y)}{d(x,y) + d(x, T_y) + d(y, T_x)} + \beta\, d(x, y)$$

for allx ,y ϵX , x ≠ y,and$0 \leq \alpha + \beta \leq 1$ then T has unique fixed point in X. See[5][6].

Definition 2.6: A sequence { x_n} in metric space (X,d) is called Cauchy sequence if for

Each$\varepsilon < 0$, there exists a positive integern_0 such that$m, n \geq n_0 \Rightarrow d(x_m, x_n) < \varepsilon$

Definition 2.7 : A complete metric space is a metric space in which every Cauchy sequence is convergent.

Definition 2.8 : A sequence { x_n} converges to x if$\lim_{n\to\infty} d(x_n, x) = 0$ in this case x is called a limit of $\{x_n\}$ and we write$x_n \to x$.

II. MAIN RESULTS

Theorem-3.1: Let T be a continuous self map, defined on a complete metric space X, satisfies the following condition ;

$$d(x_{n+1}, x_n) \leq \alpha \left[\frac{d^3(x,T_x) + d^3(x,T_y)}{1 + d^2(x,T_x)}\right] + \beta\left[d(y, T_y).d(y, T_x)\right] + \gamma\, d(x,y) + \eta[d(x, T_x) + d(y, T_y)] + \delta[d(x, T_y) + d(y, T_x)] \quad (3.1.1)$$

for all x,y,$x \neq y$and for some α,β,γ,η,δ ∈ [0,1) with (α+2β+2γ+2δ+η)<1.then T has fixed point in X.

Proof:Let $x_0$be an arbitrary point in X and we define a sequence $\{x_n\}$by means of iterates of T by setting, $T_{x_0}^n = x_n$ where n is a positive integer. If $x_n = x_{n+1}$ for some n,then x_n is affixed point of T.Taking $x_n \neq x_{n+1}$,for all n.

$d(x_{n+1},x_n) = d(T_{x_n},T_{x_{n-1}})$

$$d(x_{n+1},x_n) \le \alpha\left[\frac{d^3(x_n,T_{x_n})+d^3(x_n,T_{x_{n-1}})}{1+d^2(x_n,T_{x_n})}\right]+\beta\,[d(x_{n-1},T_{x_{n-1}}).d(x_{n-1},T_{x_n})] + \gamma\, d(x_n,x_{n-1})$$
$$+\eta[d(x_n,T_{x_n})+d(x_{n-1},T_{x_{n-1}})]+\delta\,[d(x_n,T_{x_n-1})+d(x_{n-1},T_{x_n})]$$
$$\le \alpha\left[\frac{d^3(x_n,x_{n+1})+d^3(x_n,x_n)}{1+d^2(x_n,x_{n+1})}\right]+\beta\,[d(x_{n-1},x_n).d(x_{n-1},x_{n+1})]+\gamma\, d(x_n,x_{n-1})$$
$$+\eta[d(x_n,x_{n+1})+d(x_{n-1},x_n)]+\delta\,[d(x_n,x_n)+d(x_{n-1},x_{n+1})]$$
$$\le \alpha\left[\frac{d(x_n,x_{n+1})+d(x_n,x_n)\{d^2(x_n,x_{n+1})+d^2(x_n,x_n)+d(x_n,x_{n+1})d(x_n,x_n)\}}{1+d^2(x_n,x_{n+1})}\right]$$
$$+\beta\,d(x_n,x_{n+1})+\gamma\, d(x_n,x_{n-1})+\eta[d(x_n,x_{n+1})+d(x_{n-1},x_n)]+\delta d(x_{n-1},x_{n+1})$$
$$\le \alpha\left[\frac{d(x_n,x_{n+1})d^2(x_n,x_{n+1})}{d^2(x_n,x_{n+1})}\right]+\beta[\,d(x_n,x_{n+1})]+\gamma\, d(x_n,x_{n-1})+\eta\,[d(x_n,x_{n+1})+d(x_{n-1},x_n)]$$
$$+\delta\,[d(x_{n-1},x_{n+1})]$$
$$d(x_n,x_{n+1}) \le \alpha d(x_n,x_{n+1})+\beta d(x_n,x_{n+1})+\gamma\, d(x_n,x_{n-1})+\eta[d(x_n,x_{n+1})+d(x_{n-1},x_n)]$$
$$+\delta d(x_{n-1},x_{n+1}) \qquad (3.1.2)$$
$$d(x_n,x_{n+1}) \le (\alpha+\beta+\eta+\delta)d(x_n,x_{n+1})+(\gamma+\eta+\delta)\,d(x_n,x_{n-1})$$
$$\{1-(\alpha+\beta+\eta+\delta)\}\,d(x_n,x_{n+1}) \le (\gamma+\eta+\delta)\,d(x_n,x_{n-1})$$

$$d(x_n,x_{n+1}) \le \frac{(\gamma+\eta+\delta)}{1-(\alpha+\beta+\eta+\delta)}d(x_n,x_{n-1})$$
$$\vdots$$
$$\left[\frac{(\gamma+\eta+\delta)}{1-(\alpha+\beta+\eta+\delta)}\right]^{ntimes} d(x_n,x_{n-1})$$

By the triangular inequality, we have m>n,

$$d(x_n,x_m) \le d(x_n,x_{n+1})+d(x_{n+1},x_{n+2})+++---d(x_{m-1},x_n)$$
$$\le (K^n+K^{n+1}+K^{n+2}+++--K^{m-1})\,d(x_1,x_0)$$

Where $K = \frac{(\gamma+\eta+\delta)}{1-(\alpha+\beta+\eta+\delta)} < 1$ (3.1.3)

$1-(\alpha+\beta+\eta+\delta) \le (\gamma+\eta+\delta)$ (3.1.4)

$(\alpha+\beta+2\eta+2\delta+\gamma) \le 1$ (3.1.5)

$\therefore\ d(x_n,x_m) \le \frac{K^n}{1-K}d(x_0,T_{x_0})$ (3.1.6)

So$\{x_n\}$is Cauchy sequence in X,So by completeness of X,there is apoint uϵX,such that $x_n \to u\ as\ n \to \infty$

Further,the continuity of T in X,implies

$$T(u) = \lim_{n\to\infty} Tx_n$$
$$= \lim_{n\to\infty} T_{x_n}$$
$$= \lim_{n\to\infty} x_{n+1}$$
$$= u$$

$\therefore$ u is fixed point of T in X.

Theorem3.2: Let T be the self map defined on a complete metric space (X,d) such that(3.1.1)holds.if for some positive integer p,T^p is continuous ,then T has a fixed point not unique.

Proof: We define a sequence $\{x_n\}$ as in Theorem-3.1 ,clearly it converges to some point u is (X,d). therefore its subsequence $\{x_{n_k}\}$,($n_k = k_p$) also converges to u.

$$T_u^p = T^p(\lim_{k\to\infty} x_{n_k}) = \lim_{k\to\infty} T^p_{x_{n_k}}$$

$$= \lim_{k\to\infty}(T_{x_{k+1}})$$

$$= u$$

Therefore u is a fixed point of T^p. Now ,we show that ,$T_u = u$.Let m be the smallest positive integer,s.t.$T_u^m = u, but T_u^q \neq u$,for q=1,2,3,-----m-1.

If m > 1,then by (3.1.1)

$d(T_u, u) = d(T_u, T_u^m) = d[T_u, T(T_u^{m-1})]$

$$d(u,T_u) \leq \alpha\left[\frac{d^3(u,T_u)+d^3(u,T(T_u^{m-1}))}{1+d^2(u,T_u)}\right] + \beta\left[d\left(T_u^{m-1},T(T_u^{m-1})\right).d(T_u^{m-1},T_u)\right]$$

$$+ \gamma\, d(u,T_u^{m-1}) + \eta\,[d(u,T_u) + \left(d\left(T_u^{m-1},T(T_u^{m-1})\right)\right)]$$

$$+ \delta\,[d\left(u,T(T_u^{m-1})\right) + d(T_u^{m-1},T_u)] \qquad (3.2.1)$$

$$\leq \alpha\left[\frac{d(u,T_u) + d\left(u,T(T_u^{m-1})\right)\{d^2(u,T_u) + d^2\left(u,T(T_u^{m-1})\right) + d(u,T_u).d\left(u,T(T_u^{m-1})\right)\}}{1+d^2(u,T_u)}\right]$$

$$+\beta[d(T_u^{m-1},u).d(T_u^{m-1},T_u)]+\gamma d(u,T_u^{m-1})+\eta\,[d(u,T_u) + d(T_u^{m-1},u)]$$

$$+\delta[d(u,u) + d(T_u^{m-1},T_u)]$$

$$\leq \alpha\left[\frac{\{d(u,T_u)+d(u\,u)\}\{d^2(u,T_u)+d^2(u,u)+d(u,T_u).d(u,u)\}}{1+d^2(u,T_u)}\right] + \beta[d(T_u^{m-1},u).d(T_u^{m-1},T_u)]$$

$$+\gamma d(u,T_u^{m-1}) + \eta\,[d(u,T_u) + d(T_u^{m-1},u)] + \delta\,[d(u,u) + d(T_u^{m-1},T_u)]$$

$$\leq \alpha\left[\frac{\{d\left(u,T_u\right)d^2\,(u,T_u)\}}{d^2\,(u,T_u)}\right] + \beta d(u,T_u) + \gamma\, d(u,T_u^{m-1}) + \eta\,[d(u,T_u) + d(T_u^{m-1},u)]$$

$$+ \delta\, d(T_u^{m-1},T_u)$$

$$d(u,T_u) \leq \alpha(u,T_u) + \beta d(u,T_u) + \gamma\, d(u,T_u^{m-1}) + \eta\,[d(u,T_u) + d(T_u^{m-1},u)]$$

$$+\delta\,[d(T_u^{m-1},u) + d(u,T_u)] \qquad (3.2.2)$$

$$d(u,T_u) \leq (\alpha+\beta+\eta+\delta)d(u,T_u) + (\gamma+\eta+\delta)d(u,T_u^{m-1}) \qquad (3.2.3)$$

$$d(u,T_u) \leq K d(u,T_u^{m-1}) \qquad (3.2.4)$$

Where $K = \frac{(\gamma+\eta+\delta)}{1-(\alpha+\beta+\eta+\delta)} < 1$

$$[1-(\alpha+\beta+\eta+\delta)]d(u,T_u) \leq (\gamma+\eta+\delta)d(u,T_u^{m-1})$$

$$(\alpha+\beta+2\eta+2\delta+\gamma) \leq 1 \qquad (3.2.5)$$

Thus we write $d(u,T_u,) \leq K^m d(u,T_u)$

Since $K^m < 1$

Therefore $d(u,T_u,) \leq d(u,T_u)$

Which is contradiaction.

Hence $T_u = u$,i.e. u is a fixed point of T. But T has not unique fixed point.

Theorem 3.3: Let, T_1 and , T_2 be two self maps, defined on a complete metric space (X,d) satisfying the condition ;

$$d(T_1x,T_2y) \leq \alpha\left[\frac{d^3(x_{2n},T_1x_{2n})+d^3(x_{2n},T_2x_{2n-1})}{1+d^2(x_{2n},T_1x_{2n})}\right]$$

$+\beta\left[d\left(x_{2n-1}, T_2x_{2n-1}\right).d\left(x_{2n-1}, T_1x_{2n}\right)\right] + \gamma\, d\,(x_{2n}, x_{2n-1})$

$+\eta\,[d((x_{2n}, T_1x_{2n}) + d\left(x_{2n-1}, T_2x_{2n-1}\right)]$

$+\delta\,[d\left(x_{2n}, T_2x_{2n-1}\right) + d(x_{2n-1}, T_1x_{2n})]$

for all x,y,$x \neq y$and for some α,β,γ,η,δ ∈ [0,1) with (α+2β+2γ+2δ+η)<1.then T has

fixed point in X and by Theorem (3.2) $T_1\ and T_2$ are continuous on X.

Then $T_1\ and T_2$ have a common fixed point.

Proof : We have $d(x_{2n+1}, x_{2n}) = d(T_1x_{2n}, T_2x_{2n-1})$

$$d(x_{2n+1}, x_{2n}) \leq \alpha\left[\frac{d^3(x_{2n}, T_1x_{2n})+d^3(x_{2n}, T_2x_{2n-1})}{1+d^2(x_{2n}, T_1x_{2n})}\right]$$

$$+\beta\left[d\left(x_{2n-1}, T_2x_{2n-1}\right).d\left(x_{2n-1}, T_1x_{2n}\right)\right]+ \gamma\, d\,(x_{2n}, x_{2n-1})$$

$$+\eta\,[d((x_{2n}, T_1x_{2n}) + d\left(x_{2n-1}, T_2x_{2n-1}\right)]$$

$$+\delta\,[d\left(x_{2n}, T_2x_{2n-1}\right) + d(x_{2n-1}, T_1x_{2n})](3.3.1)$$

$$\leq \alpha\left[\frac{\left\{d\left(x_{2n}, T_1x_{2n}\right) + d\left(x_{2n}, T_2x_{2n-1}\right)\right\}\left\{d^2\left(x_{2n}, T_1x_{2n}\right) + d^2\left(x_{2n}, T_2x_{2n-1}\right) + d\left(x_{2n}, T_1x_{2n}\right)d\left(x_{2n}, T_2x_{2n-1}\right)\right\}}{1 + d^2\,(x_{2n}, T_1x_{2n})}\right]$$

$+\beta\left[d\left(x_{2n-1}, x_{2n}\right).d\left(x_{2n-1}, x_{2n+1}\right)\right] + \gamma\, d\,(x_{2n}, x_{2n-1}) + \eta\,[d((x_{2n}, x_{2n+1})$

$+d\left(x_{2n-1}, x_{2n}\right)]+ \delta\,[d\left(x_{2n}, x_{2n}\right) + d(x_{2n-1}\ x_{2n+1})]$

$$\leq \alpha\left[\frac{\{d(x_{2n}, x_{2n+1})d^2(x_{2n}, x_{2n+1})\}}{d^2(x_{2n}, x_{2n+1},)}\right]+\beta\left[d\left(x_{2n}, x_{2n+1}\right)\right] + \gamma\, d\,(x_{2n}, x_{2n-1})$$

$$+\ \eta\ [d((x_{2n}, x_{2n+1}) + d\left(x_{2n-1}, x_{2n}\right)]+ \delta\,[d(x_{2n-1}, x_{2n}) + d((x_{2n}, x_{2n+1})]$$

$$\leq \alpha d(x_{2n}, x_{2n+1})+\beta d\left(x_{2n}, x_{2n+1}\right)+\gamma d(x_{2n}, x_{2n-1})+\eta[d((x_{2n}, x_{2n+1}) + d\left(x_{2n-1}, x_{2n}\right)]$$

$$+ \delta\,[d(x_{2n-1}, x_{2n}) + d((x_{2n}, x_{2n+1})] \quad (3.3.2)$$

$$d(x_{2n+1}, x_{2n}) \leq (\alpha + \beta + \eta + \delta)d\left(x_{2n}, x_{2n+1}\right)+(\gamma + \eta + \delta)d(x_{2n-1}, x_{2n}) \quad (3.3.3)$$

$$d\left(x_{2n+1}, x_{2n}\right) \leq \frac{(\gamma + \eta + \delta)}{1 - (\alpha + \beta + \eta + \delta)}\, d(x_{2n-1}, x_{2n})$$

$$d\left(x_{2n+1}, x_{2n}\right) \leq K^{2n} d((x_0, x_1)$$

Where K = $\frac{(\gamma+\eta+\delta)}{1-(\alpha+\beta+\eta+\delta)} < 1$

$$[1 - (\alpha + \beta + \eta + \delta)]d\left(x_{2n+1}, x_{2n}\right) \leq (\gamma + \eta + \delta)d((x_{2n-1}, x_{2n})$$

$$(\alpha + \beta + 2\eta + 2\,\delta + \gamma) < 1$$

Similarly we can show that

$$\left(x_{2n+1}, x_{2n+2}\right) \leq K^{2n+1} d((x_0, x_1)$$

Now it can be easily seen that $\{x_n\}$ is a Cauchy sequence. Let $x_n \to u$ then the subsequence$\{x_{n_p}\}$ also converges to u, for $n_p = 2p$.

Now $T_1T_2(u) = T_1T_2 \lim_{p\to\infty} x_{n_p}$

$=\lim_{p\to\infty} x_{n_p}+1$

= u

We now show that $T_2(u) \neq u$

If $T_2(u) \neq u$,then

d (u,T_2u) =d(T_1T_2u , T_2u)

$$d(u,T_2u) \leq \alpha\left[\frac{d^3(T_2u, T_1T_2u)+d^3(T_2u, T_2u)}{1+d^2(T_2u, T_1T_2u)}\right] + \beta[d(u, T_2u).d(u, T_1T_2u)] + \gamma\, d(T_2u, u)$$

$$+\eta\,[d((T_2u, T_1T_2u) + d(u, T_2u)] + \delta\,[d(T_2u, T_2u) + d(u, T_1T_2u) \quad (3.3.3)$$

$$\leq \alpha\left[\frac{d(T_2u,u) + d(T_2u, T_2u)\{(d^2T_2u,u) + d^2(T_2u, T_2u) + d(T_2u,u).d(T_2u, T_2u)\}}{1 + d^2(T_2u,u)}\right]$$

$+\beta[d(u, T_2u).d(u,u)] +\gamma\, d(T_2u, u) + \eta\,[d((T_2u, u) + d(u, T_2u)] + \delta\, d(T_2u, T_2u) +$

$d(u,u)]$

$$\leq \alpha\left[\frac{d(T_2u,u)\{d^2(T_2u,u)\}}{d^2(T_2u, u)}\right] +\gamma d(T_2u,u) + \eta\,[d((T_2u, u) + d(u, T_2u)] \leq \alpha d(T_2u, u)+\gamma\, d$$

$(T_2u, u) + \eta\,[d((T_2u, u) + d(u, T_2u)]$□

$$d(u,T_2u) \leq (\alpha + \gamma + 2\eta)\, d(T_2u, u) \quad (3.3.4)$$

which is contradiction.

$\therefore (\alpha + \beta + 2\eta + 2\,\delta + \gamma) < 1$,so $(\alpha + \gamma + 2\eta) \leq 1$.

Hence we have $T_2u = u$

Now $T_1T_2u = T_2u = u$

Thus u is the common fixed point of $T_1 and T_2$,But uniqueness is not possible.

Conclusion: In this paper we proof some theorems of fixed points in complete metric space.

REFERENCES:

[1]. Banach, Sur les operations dans les ensembles abstraits et leur application aux equations integrals, Fundamental Mathematics, 3(1922), 133-181.
[2]. R. Kannan, Some results on fixed points, Bull. Calcutta Math.Soc, 60 (1968) 71-76.,.Math. Moan., (1969) 76, 405-406 MR41,2487.
[3]. A.Branciari, fixed point theorems for mappings satisfying a general contractive condition of integral type,Int.J.Math.Math.Sci, 29(2002),no.531-536.
[4]. Fisher B, Publ. Inst. Math. Soc., (1976) 25, 193-194.
[5]. Jaggi D.S, Indian Journal of Pure and Applied Mathematics,(1977) 8, 223-230.
[6]. Jaggi D.S, Dass B.K, Bull. Cal. Math., (1980) 261-266.
[7]. S.K.Chatterjee , fixed point theotems, Comptes. Rend. Acad, Bulgaria Sci., (1972) 25, 727-730.
[8]. Fisher B, Sessa S, J. Univ. Kuwait Sci., (1988) 15, 9-117
[9]. Gupta V, Sharma R, Advances in Applied Science Research, (2012) 3 (3), 1266- 1270.
[10]. L.Ciric, A new fixed-point theorem for contractive mappings, Publ. Inst. Math. (Belgr.), 30 (1981) 25-27.
[11]. R. Kannan, Some results on fixed points, Bull. Calcutta math.Soc, 60 (1968) 71-76.
[12]. A.Meir, E. Keeler, A theorem on contraction mappings, J. Math. Anal. Appl, (28) (1969) 326-329.
[13]. N.Malhotra, B. Bansal, A common fixed point theorem for six weakly compatible and commuting maps in b-Metric Spaces, International Journal of Pure and Applied Mathematics, 101 (3) (2015) 325-337.
[14]. N. Malhotra, B.Bansal, Some Common Coupled Fixed Point Theorems for Generalized Contraction in b-Metric Spaces, J. Nonlinear Sci. Appl, 8 (2015) 8-16.
[15]. Emmanuel Boissard, Weierstrass Institute, Mahrenstrasse 39, 10117 Berlin, Germany.
[16]. Nathael Gozlan, University Paris-Est, LAMA (UMR 8050), UPEMLV, UPEC, CNRS, F-77454, Marne-la-Vallee, France.
[17]. Joseph Lehec, CEREMADE (UMR CNRS 7534) Universite Paris-Dauphine, Place de Lattre de Tassigny, 75016 Paris.
[18]. Chistian Leonard, Modal-X. University Paris Quest. Bat. G, 200 av. De la Republique. 92001 Nanterre, France.
[19]. Georg Menz, Stanford University, Department of Mathematics, 450 Serra Mall, Stanford, California 94305.\
[20]. Andre Schlichting, Institute for Applied Mathematics, University of Bonn, Endenicher Allee 60, D-53115 Bonn.
[21]. Asadollah Aghajani, Mujahid Abbas, Jamal Rezael Roshan, Common Fixed Point of Generalized Weak Contrac ve Mappings in Par ally Ordered G_b-metric Spaces, Filomat 28:6 (2014), 1087-1101 DOI 10.2298/FIL140 6087A.
[22]. B. C. Dhage 'Generalized metric spaces and topological structure. I,' Annalele S n ceale Universita I Al.l. Cuza, vol. 46, no. 1, pp. 3-24, (2000).
[23]. B. C. Dhage, 'On generalized metric spaces and topological structure. ll,' Pure and Applied Mathema ka Sciences, vol. 40, no. 1-2, pp. 37-41, (1994).
[24]. Aydi, H., Shatanawi, W., and Vetro, C., 'generalized weakly G-contrac on mapping in G-metric spaces,' Computers & Mathema cs with Applica ons, vol. 62, pp. 4222-4229, (2011).

[25]. Wang, S. Z., Li, B. Y., Gao, Z. M., Iseki, K., Some xed point theorems for expansion mappings, Math. Japonica. 29 (1984), 631-636.
[26]. W. Fechner, Stability of a functional inequalities associated with the Jordan-von Neumann functional equation, Aequationes Math., 71 (2006), 149-161.
[27]. W.Fechner, On some functional inequalities related to the logarithmic mean, Acta Math. Hungar., 128 (2010), 36-45.
[28]. P. Gavruta, A geralization of the Hyers-Ulam-Rassias stability of approximately additive mappings, J. Math. Anal. Appl., 184 (1994), 431-436.
[29]. T. M. Rassias, On the stability of the linear mapping in Banach spaces, Proc. Amer, Math. Soc., 72 (1978), 297-300.
[30]. D. Shin, C. Park, S. Farhadabadi, Stability and superstability of J^*-Homomorphisms and J^* -derivations for a generalized Cauchy-Jensen, J. Comput. Anal. Appl., 17 (2014), 125-134.
[31]. B. C. Dhage, "A common fixed point in the metric spaces," Bulle n of the Calculus a Mathematical society, vol. 91, no. 6, pp.475-480, (1999).
[32]. Aydi, H. Shatanawi, W., and Vetro, C., "On generalized weakly G-contrac on mapping in G-matric spaces," vol. 62, pp. 4222-4229, (2011).
[33]. Muraliraj, A., Hussain, R. J., Coincidence and Fixed Point Theorems for Expansive Maps in d-Metric Spaces, Int. Journal of Math. Analysis, Vol. 7, 2013, no. 44, 2171-2179.
[34]. B. C. Dhage, "Generalized metric space and mapping with fixed point," Bull. Calculus, Mathematical Society, vol. 84, pp. 329-336, (1992).
[35]. S. M. Ulam, A collection of the Mathematical problems, Interscience Pub. New York, (1960).

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STUDY OF FIXED POINT AND COMMON FIXED POINT WITH QUASI METRIC SPACE

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Abstract:

Quasi metrics have been used in several places in the literature on domain theory and the formal semantics of programming languages [1], [3]. In this paper we introduce the concept of generalized quasi metric (gq) space and establish some fixed point theorems in gq metric spaces.

Keywords: Generalised Quasi metric, Fixed point, Contractive Condition, CS Complete, CS Continuous, Complete Metric Space.

AMS Classification: 45H10.

2. Introduction and Preliminaries:

Pascel Hitzler[1] presented Rutten Smyth definition [2.1] for quasi metric space and applied it to semantic analysis of logic programs. In this paper we prove the gq metric version of Rutten Smyth Theorem. Rhoades[3] collected a large number of variants of Banach's Contractive conditions on self maps on a metric space and proved various implications or otherwise among them. We pick up a good number of these conditions which ultimately imply Banach condition [4]. We prove that these implications hold good for self maps on a gq metric space and prove the gq metric version of Banach's result then by deriving the gq analogue's of fixed point theorems of Rakotch, Edelstein, Kannan [5],Bianchini[6], Reich and Ciric[14]. We denote the set of non-negative real numbers by R^+ and set of natural numbers by N. EkelandVariational Principle, proved by Ivar Ekeland in 1974, see [8], turned to be one of the most powerful tools in optimization theory, differentiability of convex and Lipschitz functions, and in other areas. At the same time it lies at the origins of a plethora of variational principles with astonishing applications in mathematics, economics, biology, and other domains. The aim of the present paper is to prove a version of housdroff sequentially complete quasi-metric spaces and apply it to some fixed point theorems. It is well known that in the metric case the validity of implies the completeness of the space, a result proved by Weston [10] in (1977) and rediscovered by Sullivan [11] in (1981). The fact that the validity of Caristi's fixed point theorem (a result equivalent to Ekeland Variational Principle) implies completeness was proved by Kirk [12]. The more general case of partial metric spaces was considered by Romaguera [13]

Definition 2.1: Let binary operation $\lozenge : R^+ \times R^+ \to R^+$ satisfies the following conditions:

(I) $\lozenge$ is Associative and Commutative.

(II) $\lozenge$ is continuous w.r.t to the usual metric R^+ A few typical examples are $a \lozenge b = \max\{a, b\}$, $a \lozenge b = a + b$, $a \lozenge b = ab$, $a \lozenge b = ab + a + b$ and $a \lozenge b = \frac{ab}{\max\{a,b,1\}}$ for each $a, b \in R^+$.

In what follows we fix a binary operation $\lozenge$ that satisfies (I) and (II).

Definition 2.2: A binary operation $\lozenge$ on is R^+ said to satisfy β -property if there exists a

positive real number β such that $a \lozenge b \leq \beta \max\{a, b\}$ for every $a, b \in R^+$.

Definition 2.3: Let X be a non empty set. A generalized quasi (simply gq) metric (or * d metric) on X is a function * d :$x^2 \to R^+$ that satisfies the following conditions:

(1) () * d (x, x) = 0

(2) d(x, y) * = () * d(y, x) = 0, Implies x = y

(3) d(x, z) * ≤ d (x, y) * ◊ () * d(y ,z) for each $x, y, z \in X$. The pair (X, * d) is called a

Generalized quasi (or simply * d) metric space.

Definition 2.4: A sequence (x_n) in a gq metric space (X , * d) is a (forward) Cauchy sequence if for all $\in > 0$, there corresponds $n_\varepsilon \in N$, such that for all $n \geq m \geq n_\varepsilon$. we have * d $(x_n, x_m) < \in$. A Cauchy sequence (x_n) converges to $x \in X$.if for all $y \in X$, * d (x , y) = lim * d(x_n, y). In this case

we write lim (x_n) = x . Finally X is called CS-complete if every Cauchy sequence in X converges.

Note : Let X be a gq metric space such that ◊ satisfies β - property with β ≤ 1 ,then limits of Cauchy sequence are unique.

Definition 2.5: Let X be a gq metric space. A function f: X →X is called

(1) CS-continuous if, for all Cauchy sequences$\{x_n\}$ in X with lim x_n= x , f (x_n) is a

Cauchy sequence and lim f (x_n) = f (x).

(2) Non expanding if * d (f (x), f (y))≤ * d (x , y) for all x , y ∈ X.

(3) Contractive if there exists some 0 ≤ c <1 such that * d (f (x) , f (y)) ≤ c * d (x , y)

for all x, y ∈ X .

We present the gq metric version of housdroff sequentially theorem [1] for fixed points.

3. Main Result:

Let (Y^X ,F) be a Housdorff Sequentiacally complete quasi-gauge function space generated by a family F of pseudometric and T be a continuous mapping define on (Y^X ,F) into itself, satisfying condition :

(p T(f),T(g))(x)

$$\leq \alpha\left[\frac{p(f,T(f))(x)p(g,T(g))(x)+p(f,T(g))(x)\,p(g,T(f))(x)}{p(f,g)(x)}\right]$$

+

$$\beta\left[\frac{p(f,T(f))(x)\;p(f,T(g))(x)+p(g,T(g))(x)p(g,T(f))(x)}{p(f,g)(x)}\right]$$ +

$$\gamma[\,p(f,T(f))(x)+p(g,T(g))(x)]$$

$$+\,\delta\left[p(f,T(g))(x)+p(g,T(f))(x)\right]+\eta\,p(f,g)(x)$$

For all $f,g\in Y^X$ where $\alpha,\beta,\gamma,\delta,\eta>0$ *and* $\alpha+2\beta+2\gamma+2\delta+\eta<1$. Then T has a fixed point in (Y^X ,F).

Theorem 3.1: Let (Y^X ,F) be a Housdorff Sequentiacally complete quasi-gauge function space generated by a family F of pseudo-metric and T be a continuous mapping define on (Y^X ,F) into itself, satisfying condition :

$$d(T_x,T_y)\leq \alpha\left[\frac{d^3(x,T_x)+d^3(x,T_y)}{1+d^2(x,T_x)}\right]+\beta[d(y,T_y).d(y,T_x)]+\gamma\,d(x,y)+\eta\,[d(x,T_x)+$$

$$d(y,T_y)]+\delta\,[d(x,T_y)+d(y,T_x)] \qquad (3.1.1)$$

for all x,y , x ≠ yand for some α,β,γ, η, δ∈ [0,1) with (α + 2β + 2γ + 2δ + η) < 1.then T has fixed point in X.

Proof: Let $f_0 \in Y^X$ and { f_n } be defined by

$f_{2n+1}(x) = Tf_{2n}(x)$, ∀ $n = 1,2,3,\ldots\ldots\ldots\ldots$ and for all x in X.

Then by hypothesis of the Theorem,

We have $p(f_{2n+1},f_{2n})(x) = p(Tf_{2n},Tf_{2n-1})(x)$

$$p(f_{2n+1},f_{2n})(x)\leq \alpha\left[\frac{p^3(f_{2n},Tf_{2n})(x)+p^3(f_{2n},Tf_{2n-1})(x)}{1+p^2((f_{2n},Tf_{2n})(x)}\right]+\beta[\,p(f_{2n-1},Tf_{2n-1})(x).p$$

$$(f_{2n-1},Tf_{2n})(x)]+\gamma[p(f_{2n},f_{2n-1})(x)]+\eta\,[p(f_{2n},Tf_{2n})(x)$$

$$+p(Tf_{2n-1},f_{2n-1})(x)]+\delta\,[p(f_{2n},Tf_{2n-1})(x)+$$
$$p(f_{2n-1},Tf_{2n})(x)] \qquad (3.1.2)$$

$$\leq \alpha\left[\frac{p^3(f_{2n},f_{2n+1})(x)+p^3(f_{2n},f_{2n})(x)}{1+p^2(f_{2n},f_{2n+1})(x)}\right]+\beta\,[p(f_{2n-1},f_{2n})(x)\;.\;p$$
$$(f_{2n-1},f_{2n+1})(x)\,]$$

$$+\gamma[p(f_{2n},f_{2n-1})(x)]+\eta[p(f_{2n},f_{2n+1})(x)+$$
$$p(f_{2n-1},f_{2n})(x)]$$

$$+\,\delta\,[p(f_{2n},f_{2n})(x)+p(f_{2n-1},f_{2n+1})(x)\,]$$

$$\leq \alpha[\,p(f_{2n},f_{2n+1})(x)\,]+\beta\,[p(f_{2n},f_{2n+1})(x)]+\gamma\,[p(f_{2n},f_{2n-1})(x)]$$

$$+\;\eta\,[p(f_{2n},f_{2n+1})(x)+p(f_{2n-1},f_{2n})(x)\,]+\delta\,[\,p(f_{2n-1},f_{2n+1})(x)\,]$$

$$\leq \alpha[\,p(f_{2n},f_{2n+1})(x)\,]+\beta\,[p(f_{2n},f_{2n+1})(x)]+\gamma[p(f_{2n},f_{2n-1})(x)]$$

$$+\;\eta\,[p(f_{2n},f_{2n+1})(x)+p(f_{2n-1},f_{2n})(x)\,]+\delta\,[\,p(f_{2n-1},f_{2n})(x)+p(f_{2n},f_{2n+1})(x)\,]$$

$$p(f_{2n+1},f_{2n})(x)\leq(\alpha+\beta+\eta+\delta)p(f_{2n+1},f_{2n})(x)+(\gamma+\eta+\delta)p(f_{2n-1},f_{2n}) \qquad (3.1.3)$$

$$[1-(\alpha+\beta+\eta+\delta)]\,p(f_{2n+1},f_{2n})(x)\leq(\gamma+\eta+\delta)p(f_{2n-1},f_{2n})(x)$$

$p\left(f_{2n+1},f_{2n}\right)(x) \leq \left[\frac{(\gamma+\eta+\delta)}{1-(\alpha+\beta+\eta+\delta)}\right] p\left(f_{2n-1},f_{2n}\right)(x)$ (3.1.4)

$$1-(\alpha+\beta+\eta+\delta) < (\gamma+\eta+\delta)$$

$$(\alpha+\beta+2\eta+2\delta+\gamma) < 1$$

We have

$$p\left(f_{2n+1},f_{2n}\right)(x) \leq q\, p\left(f_{2n-1},f_{2n}\right)(x)$$

$$\vdots$$

$$q^{2n+1} p\left(f_0,f_1\right)(x)$$

Now $p\left(f_{2n+1},f_{2n}\right)(x) = \sum_{i=0}^{k} p(f_{n+i-1},f_{n+i})(x)$

$$\leq \frac{q^n}{1-q} p\left(f_0,f_1\right)(x)$$

$$\to 0 \text{ as } n \to \infty$$

Hence the sequence $\{f_n\}$ is a Cauchy sequence in Y^X. By the completeness of Y^X, there exist some f' in Y^Xsuch that

$\lim_{n\to\infty} f_n = f'$

Further the continuity of T implies.

$T(f') = T(\lim_{n\to\infty} f_n)$

$= \lim_{n\to\infty} T(f_n)$

$= \lim_{n\to\infty} f_{n+1}$

$= f'$

Thus f' is a fixed point of T.

Theorem 3.2: Let T_1 *and* T_2 are two continuous mapping on a Housdroff sequentially complete quasi- gauge function space generated by a the family F of pseudo –metric and by

(3.1.1)

$$p(T_1 f, T_2 g)(x) \leq \alpha\left[\frac{p(f,T_1 f)(x).p(g,T_2 g)(x)+p(f,T_2 g)(x).p(g,T_1 f)(x)}{p(f,g)(x)}\right] + \beta\left[\frac{p(f,T_1 f)(x).p(f,T_2 g)(x)+p(g,T_2 g)(x).p(g,T_1 f)(x)}{p(f,g)(x)}\right] + \gamma[p(f,T_1 f)(x) + p(g,T_2 g)(x)] + \delta[p(f,T_2 g)(x)+p(g,T_1 f)(x)] + \eta[p(f,g)(x)]$$

for all $f, g \in Y^X$ where $\alpha,\beta,\eta,\delta,\gamma > 0$ *and* $(\alpha+\beta+2\eta+2\delta+\gamma) < 1$.

Then T_1 *and* T_2 have a fixed point in (Y^X, F). (3.2.1)

Proof: Let $f_0 \in Y^X$ be an arbitrary element and let $\{f_n\}$ be defined as

$f_{2n+1}(x) = T_1 f_{2n}(x)$

$f_{2n}(x) = T_2 f_{2n-1}(x) \forall\, n = 1,2,3, \ldots\ldots\ldots\ldots$ and for all x in X.

Then by hypothesis of the Theorem,

We have

$p(f_{2n+1},f_{2n})(x) = p(T_1 f_{2n}(x), T_2 f_{2n-1})(x)$

$$p(f_{2n+1},f_{2n})(x) \leq \alpha\left[\frac{p^3(f_{2n},T_1 f_{2n})(x)+p^3(f_{2n},T_2 f_{2n-1})(x)}{1+p^2(f_{2n},T_1 f_{2n})(x)}\right] + \beta[p(f_{2n-1},T_2 f_{2n-1})(x).p(f_{2n-1},T_1 f_{2n})(x)] + \gamma[p(f_{2n},f_{2n-1})(x)] + \eta[p(f_{2n},T_1 f_{2n})(x) + p(f_{2n-1},T_2 f_{2n-1})(x)] + \delta[p(f_{2n},T_2 f_{2n-1})(x) + p(f_{2n-1},T_1 f_{2n})(x)] \quad (3.2.2)$$

$$\leq \alpha\left[\frac{p^3(f_{2n},f_{2n+1})(x)+p^3(f_{2n},f_{2n})(x)}{1+p^2(f_{2n},f_{2n+1})(x)}\right] + \beta[p(f_{2n-1},f_{2n})(x).p(f_{2n-1},f_{2n+1})(x)] + \gamma[p(f_{2n},f_{2n-1})(x)] + \eta[p(f_{2n},f_{2n+1})(x) + p(f_{2n-1},f_{2n})(x)] + \delta[p(f_{2n},f_{2n})(x) + p(f_{2n-1},f_{2n+1})(x)]$$

$$\leq \alpha[p(f_{2n},f_{2n+1})(x)] + \beta[p(f_{2n},f_{2n+1})(x)] + \gamma[p(f_{2n},f_{2n-1})(x)] + \eta[p(f_{2n},f_{2n+1})(x) + p(f_{2n-1},f_{2n})(x)] + \delta[p(f_{2n-1},f_{2n+1})(x)]$$

$$\leq \alpha[p(f_{2n},f_{2n+1})(x)] + \beta[p(f_{2n},f_{2n+1})(x)] + \gamma[p(f_{2n},f_{2n-1})(x)] + \eta[p(f_{2n},f_{2n+1})(x) + p(f_{2n-1},f_{2n})(x)] + \delta[p(f_{2n-1},f_{2n})(x) + p(f_{2n},f_{2n+1})(x)]$$

$p \quad (f_{2n+1}, f_{2n}) \quad (x) \leq (\alpha+\beta+\eta+\delta)p\,(f_{2n+1}, f_{2n})(x) + (\gamma+\eta+\delta)p\,(f_{2n-1}, f_{2n})$ (3.2.3)

$[1-(\alpha+\beta+\eta+\delta)]\,p\,(f_{2n+1}, f_{2n})(x) \leq (\gamma+\eta+\delta)p\,(f_{2n-1}, f_{2n})(x)$

$p\,(f_{2n+1}, f_{2n})(x) \leq \left[\frac{(\gamma+\eta+\delta)}{1-(\alpha+\beta+\eta+\delta)}\right] p\,(f_{2n-1}, f_{2n})(x)$ (3.2.4)

$$\frac{(\gamma+\eta+\delta)}{1-(\alpha+\beta+\eta+\delta)} = q < 1$$

$1-(\alpha+\beta+\eta+\delta)] < (\gamma+\eta+\delta)$

$\therefore (\alpha+\beta+2\eta+2\delta+\gamma) < 1$

We have $p\,(f_{2n+1}, f_{2n})(x) \leq q\,p(f_{2n-1}, f_{2n})(x)$

$$\vdots$$

$$q^{2n+1}p(f_0, f_1)(x)$$

Now $p\,(f_{2n+1}, f_{2n})(x) = \sum_{i=0}^{k} p(f_{n+i-1}, f_{n+i})(x)\, Y^X$

$$\leq \frac{q^n}{1-q}\, p((f_0, f_1)(x)$$

$$\to 0 \text{ as } n \to \infty$$

Hence the sequence $\{f_n\}$ is a Cauchy sequence in Y^X. By the completeness of Y^X, there exist some f' in Y^Xsuch that

$\lim_{n\to\infty} f_n = z$

Further the continuity of T_1 implies,

$T_1(z) = T(\lim_{n\to\infty} f_n)$

$= \lim_{n\to\infty} T(f_n)$

$= z$

Thus z is a fixed point of T_1.

Similarly z is a fixed point of T_2.

Theorem 3.3: Let E ,F and T are three continuous mappings onto a housdroff sequentially complete quasi-gauge function space generated by the family p of pseudo-metric and by

ET = TE , FT = TE and $E\,(Y^X) \subset T(Y^X)$, $F\,(Y^X) \subset T(Y^X)$ (3.3.1)

$$p(Ef, Fg)(x) \leq \alpha\left[\frac{p(Tf,Ef)(x)\,p(Tg,Fg)(x)+p(Tf,Fg)(x).p(Tg,Ef)(x)}{p(Tf,Tg)(x)}\right]$$

$$+\beta\left[\frac{p(Tf,Ef)(x).p(Tf,Fg)(x)+p(Tg,Fg)(x).p(Tg,Ef)(x)}{p(Tf,Tg)(x)}\right]+\gamma\,[p(Tf,Ef)(x)+p(Tg,Fg)(x)]$$

$$+\delta[\,p(Tf,Fg)(x)+p(Tg,Ef)(x)]\;+\;\eta\,[p(Tf,Tg)(x)] \quad (3.3.2)$$

for all $f, g \in Y^X$ where $\alpha, \beta, \eta, \delta, \gamma > 0$ and $(\alpha+\beta+2\eta+2\delta+\gamma) < 1$. Then E ,F and T have

a fixed point in Y^X.

Proof: Let $f_0 \in Y^X$ and $E\,(Y^X) \subset T(Y^X)$ and $F\,(Y^X) \subset T(Y^X)$

We define a sequence $\{Tf_n\}$ as follows (3.1.1)

$Tf_{2n+1}(x) = Ef_{2n}(x)$

$Tf_{2n}(x) = Ff_{2n-1}(x)$, $\forall\, n = 1,2,3, \ldots\ldots\ldots\ldots$ and for all x in X.

Now by hypothesis of the Theorem, we have

$p\,(Tf_{2n+1}, Tf_{2n})\,(x) = p\,(Ef_{2n}(x), Ff_{2n-1})\,(x)$

$$p\;(Tf_{2n+1}, Tf_{2n})\;(x) \leq \alpha\,[\frac{p^3(Tf_{2n}, Ef_{2n})(x)+p^3(Tf_{2n}, Ff_{2n-1})(x)}{1+p^2(Tf_{2n}, Ef_{2n})(x)}]$$

$$+\beta\,[p\,(Tf_{2n-1}, Ff_{2n-1})\,(x)\,.\,p\;(Tf_{2n-1}, Ef_{2n})\;(x)] \quad +\gamma\,[p(Tf_{2n}, Tf_{2n-1})\,(x)] \quad + [p\,(Tf_{2n}, Ef_{2n})(x) + (Tf_{2n-1}, Ff_{2n-1})(x)] +$$

$$\delta\,[p\,(Tf_{2n}, Ff_{2n-1})(x) + p\,(Tf_{2n-1}, Ef_{2n})(x)\,] \quad (3.3.3)$$

$$\leq \alpha\left[\frac{\{p(Tf_{2n}, Tf_{2n+1})(x) + p(Tf_{2n}, Tf_{2n})(x)\}\{p^2(Tf_{2n}, Tf_{2n+1})(x) +}{1+p^2(Tf_{2n},}\right.$$

$$+\beta\,[p(Tf_{2n}, Tf_{2n+1})(x)]+\gamma[\,p(Tf_{2n}, Tf_{2n-1}(x)] + \eta\,[p\,(Tf_{2n}, Tf_{2n+1})(x) + p\,(Tf_{2n-1}, Tf_{2n})(x)]$$

$$+\delta\,[\,p\,(Tf_{2n-1}, Tf_{2n+1})(x)]$$

$$\leq \alpha\left[\frac{\{p(Tf_{2n}, Tf_{2n+1})(x)\}\{p^2(Tf_{2n}, Tf_{2n+1})(x)\}}{p^2(Tf_{2n}, Tf_{2n+1})(x)}\right] + \beta\,[p(Tf_{2n}, Tf_{2n+1})(x)] + \gamma[(p(Tf_{2n}, Tf_{2n-1})(x)] +$$

$\eta\,[p\,(Tf_{2n},Tf_{2n+1})(x) + p\,(Tf_{2n-1},Tf_{2n})(x)] +$
$\delta\,[\,p\,(Tf_{2n-1},Tf_{2n})(x) + p\,(Tf_{2n},Tf_{2n+1})(x)\,]$

$\leq \alpha\,[\,p\,(Tf_{2n},Tf_{2n+1})(x)] + \beta\,[p(Tf_{2n},Tf_{2n+1})(x)]$
$+\gamma[(p(\,Tf_{2n},Tf_{2n-1}\,)(x)]$

$+\eta[p(Tf_{2n},Tf_{2n+1})(x) + p(Tf_{2n-1},Tf_{2n})(x)]$
$+\delta[p(Tf_{2n-1},Tf_{2n})(x) + p(Tf_{2n},Tf_{2n+1})(x)]$

$p\,(Tf_{2n+1},Tf_{2n})(x) \leq (\alpha+\beta+\eta+\delta)\,p(Tf_{2n+1},Tf_{2n})(x) +$
$(\gamma+\eta+\delta)p\,(Tf_{2n},f_{2n-1})(x)$

$[1-(\alpha+\beta+\eta+\delta)]\,p\,(Tf_{2n+1},Tf_{2n})(x) \leq (\gamma+\eta+$
$\delta)p\,(Tf_{2n-1},Tf_{2n})(x)$

$$p\,(Tf_{2n+1},Tf_{2n})(x) \leq \frac{(\gamma+\eta+\delta)}{1-(\alpha+\beta+\eta+\delta)}\,p\,(Tf_{2n-1},Tf_{2n})(x)$$

$$\frac{(\gamma+\eta+\delta)}{1-(\alpha+\beta+\eta+\delta)} = q < 1$$

$1-(\alpha+\beta+\eta+\delta)] < (\gamma+\eta+\delta)$

$(\alpha+\beta+2\eta+2\delta+\gamma) < 1$

We have $p\,(Tf_{2n+1},Tf_{2n})(x) \leq q\,p(Tf_{2n-1},Tf_{2n})(x)$

$$\vdots$$

$$q^{2n+1}p(Tf_0,Tf_1)(x)$$

Now $p\,(Tf_{n+k},Tf_{2n})(x) = \sum_{i=0}^{k} p\,(Tf_{n+i-1},Tf_{n+i})(x)$

$$< \frac{q^n}{1-q}\,p((Tf_0,Tf_1)(x)$$

$$\to 0 \text{ as } n \to \infty$$

Hence the sequence $\{f_n\}$ is a Cauchy sequence in Y^X. By the completeness of Y^X, $\{f_n\}$ converges to a point z in Y^X.

Conclusion:

In this paper we investigations the existence of fixed point in quasi metric space with completeness.

References:

[1] P.Hitzler, Generalized metrics and topology in logic programming semantics, Ph. D Thesis, School of Mathematics, Applied Mathematics and Statistics, National University Ireland, University College Cork,(2001).

[2] S.G. Mathews, Metric domains for completeness, Technical report 76Department of Computer Science, University of Warwick, U.K, Ph.D Thesis (1985).

[3] B.E.Rhoades,A comparison of various definitions of contractive mappings, Trans of the Amer. Math. Society Vol .226(1977) P.P.257-290.

[4] Banach, sur les operations dans les ensembles abstraitsetleur application aux equations integrals, fundamental, Mathematics 3(1922),133-181.

[5] R .Kannan, Some results on fixed points, Bull .CalcuttaMath.Soc,60(1968),71-76,Math. Moan.,(1969),76,405-406 MR41,2487.

[6] A. Branciari, fixed point theorems for mappings satisfying a general contractive conditionof Integraltype,Int.J.Math.Sci,29(2002),no.531-536.

[7] I. Ekeland, Nonconvex minimization problems, Bull. Amer. Math. Soc. (N.S.)1 (3) (1979) 443–474.

[8] E. Alemany, S. Romaguera, On right K -sequentially complete quasi-metric spaces, ActaMath. Hungar. 75 (3) (1997) 267–278.

[9] A. Kristály, V.D. Radulescu, Cs. Varga, Variational Principles in Mathematical Physics, Geometry, and Economics – Qualitative Analysis of Nonlinear ˘ Equations and Unilateral

[10] Problems, Encyclopedia Math. Appl. Cambridge University Press, Cambridge, (2010)10 F. Sullivan, A characterization of complete metric spaces, Proc. Amer.Math.Soc.83 (2) (1981) 345–346.

[11] W.A. Kirk, Caristi's fixed point theorem and metric convexity, Colloq. Math. 36 (1) (1976) 81–86.

[12] S. Romaguera, A Kirk type characterization of completeness for partial metric spaces, Fixed Point Theory Appl. (2010) Art. ID 493298, 6 pp.

[13] D. Azé, J.-N. Corvellec, On some variational properties of metric spaces, J. Fixed Point Theory Appl. 5 (1) (2009) 185–200.

[14] H. Brézis, F.E. Browder, A general principle on ordered sets in nonlinear functional analysis, Advances in Math. 21 (3) (1976) 355–364.

[15] J. Caristi, Fixed point theorems for mappings satisfying inwardness conditions, Trans.Amer. Math. Soc. 215 (1976) 241–251.

[16] J. Caristi, W.A. Kirk, Geometric fixed point theory and inwardness conditions, in: TheGeometry of Metric and Linear Spaces, Proc. Conf., Michigan State Univ., East Lansing, Mich., (1974) in Lecture Notes in Math., vol. 490, Springer, Berlin,(1975) pp. 74–83.

[17] S.A. Chen, W. Li, S.P. Tian, Z.Y. Mao, On optimization problems in quasi-metric spaces, In Proc. 5th

International Conf. on Machine Learning and Cybernetics, Dalian, 13–16 ,Aug. (2006) pp. 865–870.

[18] S. Chen, S. Tian, Z. Mao, On Caristi's fixed point theorem in quasi-metric spaces, Dyn.Contin. Discrete Impuls. Syst. Ser. A Math. Anal. 13A (Part 3, suppl.) (2006) 1150–1157

[19] S.A. Chen, W. Li, D. Zou, S.B. Chen, Fixed point theorems in quasi-metric spaces, in: Proc. 6th International Conf. on Machine Learning and Cybernetics, Hong Kong, 19–22 Aug. (2007) pp. 2499–2504.

[20] L.Ciri ´ c, Periodic and fixed point theorems in a quasi-metric space, J. Austral. Math. Soc. Ser. A 54 (1) (1993) 80–85.

[21] S. Cobzas¸ Asymmetric locally convex spaces, Int. J. Math. Math. Sci. (16) (2005) 2585–2608.

[22] S. Cobza¸s, Compact and precompact sets in asymmetric locally convex spaces, Topology Appl.156 (9) (2009) 1620–1629

[23] S.Cobza¸s, Functional analysis inasymmetric normed spaces, arXiv:1006.1175v1, (2010).

[24] I. Ekeland, On the variational principle, J. Math. Anal. Appl. 47 (1974) 324–353.

[25] P. Fletcher, W.F. Lindgren, Quasi-Uniform Spaces, M. Dekker, New York, (1982).

[26] L.M. García-Raffi, S. Romaguera, E.A. Sánchez-Pérez, On Hausdorff asymmetric normed linear spaces, Houston J. Math. 29 (3) (2003) 717–728.

[27] L.M. García-Raffi, S. Romaguera, E.A. Sánchez-Pérez, Weak topologies on asymmetric normed linear spaces and non-asymptotic criteria in the theory of complexity analysis of algorithms, J. Anal. Appl. 2 (3) (2004) 125–138.

[28] T.L. Hicks, Fixed point theorems for quasi-metric spaces, Math. Japon. 33 (1988) 231–236.

[29] J.C. Kelly, Bitopological spaces, Proc. London Math. Soc. 13 (1963) 71–89.

[30] H.-P.A. Künzi, M. Mrševic, I.L. Reilly, M.K. Vamanamurthy, Convergence, precompactness and symmetry in quasi-uniform spaces, Math. Japon. 38 (1993)´ 239–253.

[31] A.C.G. Mennucci, On asymmetric distances, preprint, 2nd version 2007 (1st version 2004) bothavailable at http://cvgmt.sns.it/people/mennucci.

[32] I.L. Reilly, P.V. Subrahmanyam, M.K. Vamanamurthy, Cauchy sequences in quasi-pseudo-metric spaces, Monatsh. Math. 93 (1982) 127–140.

[33] S. Romaguera, Left K-completeness in quasi-metric spaces, Math. Nachr. 157 (1992) 15–23.

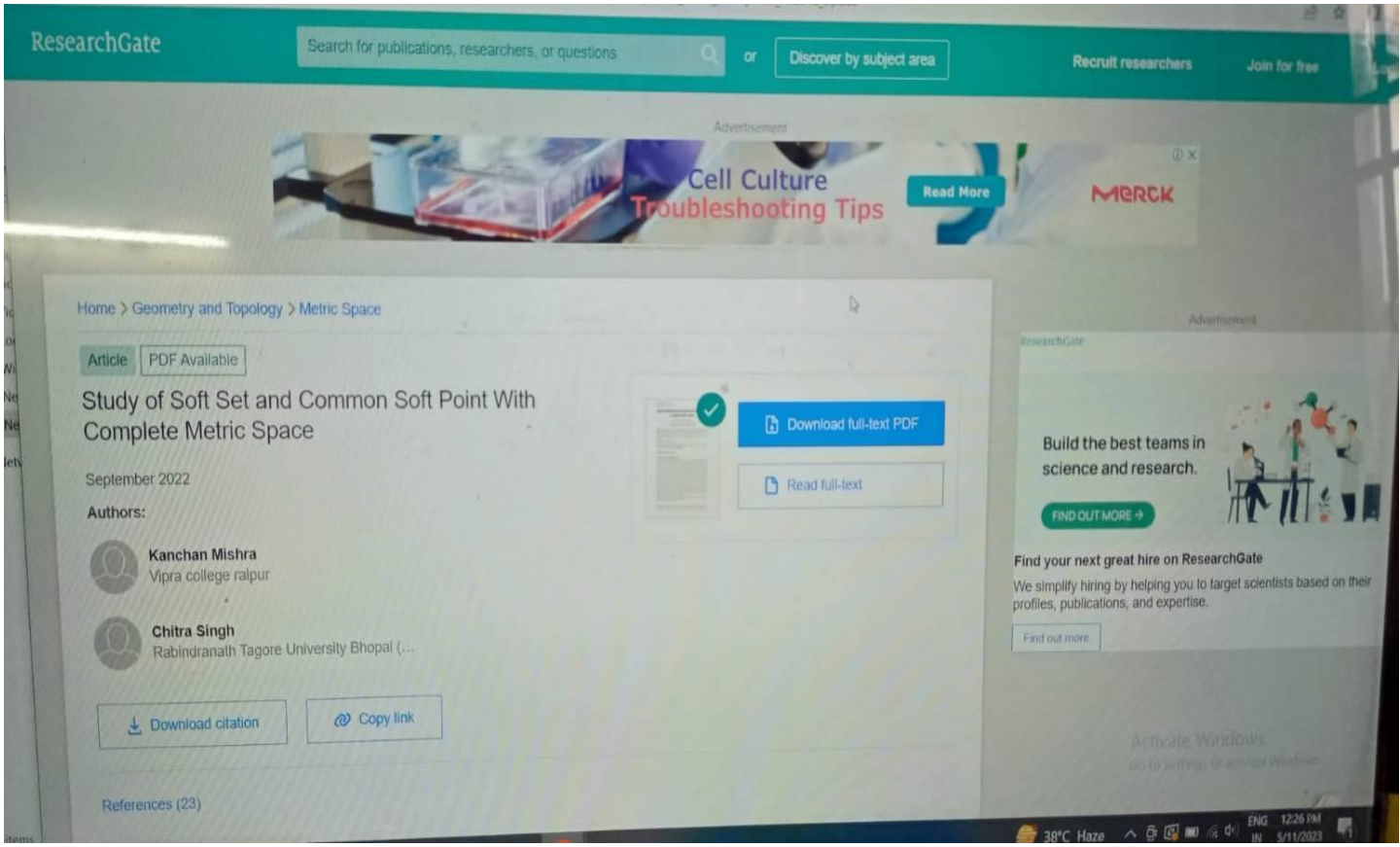
[34] S. Romaguera, A. Gutiérrez, A note on Cauchy sequences in quasipseudometric spaces, Glas. Mat.Ser. III 21(41) (1) (1986) 191–200.

[35] S. Romaguera, M. Schell kens, Quasi-metric properties of complexity spaces, Topology

[36] S. Romaguera, M. Schell kens, Duality and quasi-normability for complexity spaces, Appl. Gen .Topol. 3 (1) (2002) 91–112.

[37] R.A. Stoltenberg, On quasi-metric spaces, Duke Math. J. 36 (1969) 65–71.

[38] J. Sh. Ume, A minimization theorem in quasi-metric spaces and its applications, Int. J. Math. Math. Sci. 31 (7) (2000).


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STUDY OF FIXED POINT AND COMMON FIXED POINT WITH QUASI METRIC SPACE

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Abstract:

Quasi metrics have been used in several places in the literature on domain theory and the formal semantics of programming languages [1], [3]. In this paper we introduce the concept of generalized quasi metric (gq) space and establish some fixed point theorems in gq metric spaces.

Keywords: Generalised Quasi metric, Fixed point, Contractive Condition, CS Complete, CS Continuous, Complete Metric Space.

AMS Classification: 45H10.

2. Introduction and Preliminaries:

Pascel Hitzler[1] presented Rutten Smyth definition [2.1] for quasi metric space and applied it to semantic analysis of logic programs. In this paper we prove the gq metric version of Rutten Smyth Theorem. Rhoades[3] collected a large number of variants of Banach's Contractive conditions on self maps on a metric space and proved various implications or otherwise among them. We pick up a good number of these conditions which ultimately imply Banach condition [4]. We prove that these implications hold good for self maps on a gq metric space and prove the gq metric version of Banach's result then by deriving the gq analogue's of fixed point theorems of Rakotch, Edelstein, Kannan [5],Bianchini[6], Reich and Ciric[14]. We denote the set of non-negative real numbers by R^+ and set of natural numbers by N. EkelandVariational Principle, proved by Ivar Ekeland in 1974, see [8], turned to be one of the most powerful tools in optimization theory, differentiability of convex and Lipschitz functions, and in other areas. At the same time it lies at the origins of a plethora of variational principles with astonishing applications in mathematics, economics, biology, and other domains. The aim of the present paper is to prove a version of housdroff sequentially complete quasi-metric spaces and apply it to some fixed point theorems. It is well known that in the metric case the validity of implies the completeness of the space, a result proved by Weston [10] in (1977) and rediscovered by Sullivan [11] in (1981). The fact that the validity of Caristi's fixed point theorem (a result equivalent to Ekeland Variational Principle) implies completeness was proved by Kirk [12]. The more general case of partial metric spaces was considered by Romaguera [13]

Definition 2.1: Let binary operation $\lozenge : R^+ \times R^+ \to R^+$ satisfies the following conditions:

(I) $\lozenge$ is Associative and Commutative.

(II) $\lozenge$ is continuous w.r.t to the usual metric R^+ A few typical examples are $a \lozenge b = \max\{a, b\}$, $a \lozenge b = a + b$, $a \lozenge b = ab$, $a \lozenge b = ab + a + b$ and $a \lozenge b = \frac{ab}{\max\{a,b,1\}}$ for each $a, b \in R^+$.

In what follows we fix a binary operation $\lozenge$ that satisfies (I) and (II).

Definition 2.2: A binary operation $\lozenge$ on is R^+ said to satisfy β -property if there exists a

positive real number β such that $a \lozenge b \leq \beta \max\{a, b\}$ for every $a, b \in R^+$.

Definition 2.3: Let X be a non empty set. A generalized quasi (simply gq) metric (or * d metric) on X is a function * d :$x^2 \to R^+$ that satisfies the following conditions:

(1) () * d (x, x) = 0

(2) d(x, y) * = () * d(y, x) = 0, Implies x = y

(3) d(x, z) * ≤ d (x, y) * ◊ () * d(y ,z) for each $x, y, z \in X$. The pair (X, * d) is called a

Generalized quasi (or simply * d) metric space.

Definition 2.4: A sequence (x_n) in a gq metric space (X , * d) is a (forward) Cauchy sequence if for all $\in > 0$, there corresponds $n_\varepsilon \in N$, such that for all $n \geq m \geq n_\varepsilon$. we have * d $(x_n, x_m) < \in$. A Cauchy sequence (x_n) converges to $x \in X$.if for all $y \in X$, * d (x , y) = lim * d(x_n, y). In this case

we write lim (x_n) = x . Finally X is called CS-complete if every Cauchy sequence in X converges.

Note : Let X be a gq metric space such that ◊ satisfies β - property with β ≤ 1 ,then limits of Cauchy sequence are unique.

Definition 2.5: Let X be a gq metric space. A function f: X →X is called

(1) CS-continuous if, for all Cauchy sequences$\{x_n\}$ in X with lim x_n= x , f (x_n) is a

Cauchy sequence and lim f (x_n) = f (x).

(2) Non expanding if * d (f (x), f (y))≤ * d (x , y) for all x , y ∈ X.

(3) Contractive if there exists some 0 ≤ c <1 such that * d (f (x) , f (y)) ≤ c * d (x , y)

for all x, y ∈ X .

We present the gq metric version of housdroff sequentially theorem [1] for fixed points.

3. Main Result:

Let (Y^X ,F) be a Housdorff Sequentiacally complete quasi-gauge function space generated by a family F of pseudometric and T be a continuous mapping define on (Y^X ,F) into itself, satisfying condition :

(p T(f),T(g))(x)

$$\leq \alpha\left[\frac{p(f,T(f))(x)p(g,T(g))(x)+p(f,T(g))(x)\,p(g,T(f))(x)}{p(f,g)(x)}\right]$$

+

$$\beta\left[\frac{p(f,T(f))(x)\;p(f,T(g))(x)+p(g,T(g))(x)p(g,T(f))(x)}{p(f,g)(x)}\right]$$ +

$$\gamma[\,p(f,T(f))(x)+p(g,T(g))(x)]$$

$$+\,\delta\left[p(f,T(g))(x)+p(g,T(f))(x)\right]+\eta\,p(f,g)(x)$$

For all $f,g\in Y^X$ where $\alpha,\beta,\gamma,\delta,\eta>0$ *and* $\alpha+2\beta+2\gamma+2\delta+\eta<1$. Then T has a fixed point in (Y^X ,F).

Theorem 3.1: Let (Y^X ,F) be a Housdorff Sequentiacally complete quasi-gauge function space generated by a family F of pseudo-metric and T be a continuous mapping define on (Y^X ,F) into itself, satisfying condition :

$$d(T_x,T_y)\leq \alpha\left[\frac{d^3(x,T_x)+d^3(x,T_y)}{1+d^2(x,T_x)}\right]+\beta[d(y,T_y).d(y,T_x)]+\gamma\,d(x,y)+\eta\,[d(x,T_x)+$$

$$d(y,T_y)]+\delta\,[d(x,T_y)+d(y,T_x)] \qquad (3.1.1)$$

for all x,y , x ≠ yand for some α,β,γ, η, δ∈ [0,1) with (α + 2β + 2γ + 2δ + η) < 1.then T has fixed point in X.

Proof: Let $f_0 \in Y^X$ and { f_n } be defined by

$f_{2n+1}(x) = Tf_{2n}(x)$, ∀ $n = 1,2,3,\ldots\ldots\ldots\ldots$ and for all x in X.

Then by hypothesis of the Theorem,

We have $p(f_{2n+1},f_{2n})(x) = p(Tf_{2n},Tf_{2n-1})(x)$

$$p(f_{2n+1},f_{2n})(x)\leq \alpha\left[\frac{p^3(f_{2n},Tf_{2n})(x)+p^3(f_{2n},Tf_{2n-1})(x)}{1+p^2((f_{2n},Tf_{2n})(x)}\right]+\beta[\,p(f_{2n-1},Tf_{2n-1})(x).p$$

$$(f_{2n-1},Tf_{2n})(x)]+\gamma[p(f_{2n},f_{2n-1})(x)]+\eta\,[p(f_{2n},Tf_{2n})(x)$$

$$+p(Tf_{2n-1},f_{2n-1})(x)]+\delta\,[p(f_{2n},Tf_{2n-1})(x)+$$
$$p(f_{2n-1},Tf_{2n})(x)] \qquad (3.1.2)$$

$$\leq \alpha\left[\frac{p^3(f_{2n},f_{2n+1})(x)+p^3(f_{2n},f_{2n})(x)}{1+p^2(f_{2n},f_{2n+1})(x)}\right]+\beta\,[p(f_{2n-1},f_{2n})(x)\;.\;p$$
$$(f_{2n-1},f_{2n+1})(x)\,]$$

$$+\gamma[p(f_{2n},f_{2n-1})(x)]+\eta[p(f_{2n},f_{2n+1})(x)+$$
$$p(f_{2n-1},f_{2n})(x)]$$

$$+\,\delta\,[p(f_{2n},f_{2n})(x)+p(f_{2n-1},f_{2n+1})(x)\,]$$

$$\leq \alpha[\,p(f_{2n},f_{2n+1})(x)\,]+\beta\,[p(f_{2n},f_{2n+1})(x)]+\gamma\,[p(f_{2n},f_{2n-1})(x)]$$

$$+\;\eta\,[p(f_{2n},f_{2n+1})(x)+p(f_{2n-1},f_{2n})(x)\,]+\delta\,[\,p(f_{2n-1},f_{2n+1})(x)\,]$$

$$\leq \alpha[\,p(f_{2n},f_{2n+1})(x)\,]+\beta\,[p(f_{2n},f_{2n+1})(x)]+\gamma[p(f_{2n},f_{2n-1})(x)]$$

$$+\;\eta\,[p(f_{2n},f_{2n+1})(x)+p(f_{2n-1},f_{2n})(x)\,]+\delta\,[\,p(f_{2n-1},f_{2n})(x)+p(f_{2n},f_{2n+1})(x)\,]$$

$$p(f_{2n+1},f_{2n})(x)\leq(\alpha+\beta+\eta+\delta)p(f_{2n+1},f_{2n})(x)+(\gamma+\eta+\delta)p(f_{2n-1},f_{2n}) \qquad (3.1.3)$$

$$[1-(\alpha+\beta+\eta+\delta)]\,p(f_{2n+1},f_{2n})(x)\leq(\gamma+\eta+\delta)p(f_{2n-1},f_{2n})(x)$$

$p\left(f_{2n+1},f_{2n}\right)(x) \leq \left[\frac{(\gamma+\eta+\delta)}{1-(\alpha+\beta+\eta+\delta)}\right] p\left(f_{2n-1},f_{2n}\right)(x)$ (3.1.4)

$$1-(\alpha+\beta+\eta+\delta) < (\gamma+\eta+\delta)$$

$$(\alpha+\beta+2\eta+2\delta+\gamma) < 1$$

We have

$$p\left(f_{2n+1},f_{2n}\right)(x) \leq q\, p\left(f_{2n-1},f_{2n}\right)(x)$$

$$\vdots$$

$$q^{2n+1} p\left(f_0,f_1\right)(x)$$

Now $p\left(f_{2n+1},f_{2n}\right)(x) = \sum_{i=0}^{k} p(f_{n+i-1},f_{n+i})(x)$

$$\leq \frac{q^n}{1-q} p\left(f_0,f_1\right)(x)$$

$$\to 0 \text{ as } n \to \infty$$

Hence the sequence $\{f_n\}$ is a Cauchy sequence in Y^X. By the completeness of Y^X, there exist some f' in Y^Xsuch that

$\lim_{n\to\infty} f_n = f'$

Further the continuity of T implies.

$T(f') = T(\lim_{n\to\infty} f_n)$

$= \lim_{n\to\infty} T(f_n)$

$= \lim_{n\to\infty} f_{n+1}$

$= f'$

Thus f' is a fixed point of T.

Theorem 3.2: Let T_1 *and* T_2 are two continuous mapping on a Housdroff sequentially complete quasi- gauge function space generated by a the family F of pseudo –metric and by

(3.1.1)

$$p(T_1 f, T_2 g)(x) \leq \alpha\left[\frac{p(f,T_1 f)(x).p(g,T_2 g)(x)+p(f,T_2 g)(x).p(g,T_1 f)(x)}{p(f,g)(x)}\right] + \beta\left[\frac{p(f,T_1 f)(x).p(f,T_2 g)(x)+p(g,T_2 g)(x).p(g,T_1 f)(x)}{p(f,g)(x)}\right] + \gamma[p(f,T_1 f)(x) + p(g,T_2 g)(x)] + \delta[p(f,T_2 g)(x)+p(g,T_1 f)(x)] + \eta[p(f,g)(x)]$$

for all $f, g \in Y^X$ where $\alpha,\beta,\eta,\delta,\gamma > 0$ *and* $(\alpha+\beta+2\eta+2\delta+\gamma) < 1$.

Then T_1 *and* T_2 have a fixed point in (Y^X, F). (3.2.1)

Proof: Let $f_0 \in Y^X$ be an arbitrary element and let $\{f_n\}$ be defined as

$f_{2n+1}(x) = T_1 f_{2n}(x)$

$f_{2n}(x) = T_2 f_{2n-1}(x) \forall\, n = 1,2,3, \ldots\ldots\ldots\ldots$ and for all x in X.

Then by hypothesis of the Theorem,

We have

$p(f_{2n+1},f_{2n})(x) = p(T_1 f_{2n}(x), T_2 f_{2n-1})(x)$

$$p(f_{2n+1},f_{2n})(x) \leq \alpha\left[\frac{p^3(f_{2n},T_1 f_{2n})(x)+p^3(f_{2n},T_2 f_{2n-1})(x)}{1+p^2(f_{2n},T_1 f_{2n})(x)}\right] + \beta[p(f_{2n-1},T_2 f_{2n-1})(x).p(f_{2n-1},T_1 f_{2n})(x)] + \gamma[p(f_{2n},f_{2n-1})(x)] + \eta[p(f_{2n},T_1 f_{2n})(x) + p(f_{2n-1},T_2 f_{2n-1})(x)] + \delta[p(f_{2n},T_2 f_{2n-1})(x) + p(f_{2n-1},T_1 f_{2n})(x)] \quad (3.2.2)$$

$$\leq \alpha\left[\frac{p^3(f_{2n},f_{2n+1})(x)+p^3(f_{2n},f_{2n})(x)}{1+p^2(f_{2n},f_{2n+1})(x)}\right] + \beta[p(f_{2n-1},f_{2n})(x).p(f_{2n-1},f_{2n+1})(x)] + \gamma[p(f_{2n},f_{2n-1})(x)] + \eta[p(f_{2n},f_{2n+1})(x) + p(f_{2n-1},f_{2n})(x)] + \delta[p(f_{2n},f_{2n})(x) + p(f_{2n-1},f_{2n+1})(x)]$$

$$\leq \alpha[p(f_{2n},f_{2n+1})(x)] + \beta[p(f_{2n},f_{2n+1})(x)] + \gamma[p(f_{2n},f_{2n-1})(x)] + \eta[p(f_{2n},f_{2n+1})(x) + p(f_{2n-1},f_{2n})(x)] + \delta[p(f_{2n-1},f_{2n+1})(x)]$$

$$\leq \alpha[p(f_{2n},f_{2n+1})(x)] + \beta[p(f_{2n},f_{2n+1})(x)] + \gamma[p(f_{2n},f_{2n-1})(x)] + \eta[p(f_{2n},f_{2n+1})(x) + p(f_{2n-1},f_{2n})(x)] + \delta[p(f_{2n-1},f_{2n})(x) + p(f_{2n},f_{2n+1})(x)]$$

$p \quad (f_{2n+1}, f_{2n}) \quad (x) \leq (\alpha+\beta+\eta+\delta)p\,(f_{2n+1}, f_{2n})(x) + (\gamma+\eta+\delta)p\,(f_{2n-1}, f_{2n})$ (3.2.3)

$[1-(\alpha+\beta+\eta+\delta)]\,p\,(f_{2n+1}, f_{2n})(x) \leq (\gamma+\eta+\delta)p\,(f_{2n-1}, f_{2n})(x)$

$p\,(f_{2n+1}, f_{2n})(x) \leq \left[\frac{(\gamma+\eta+\delta)}{1-(\alpha+\beta+\eta+\delta)}\right] p\,(f_{2n-1}, f_{2n})(x)$ (3.2.4)

$$\frac{(\gamma+\eta+\delta)}{1-(\alpha+\beta+\eta+\delta)} = q < 1$$

$1-(\alpha+\beta+\eta+\delta)] < (\gamma+\eta+\delta)$

$\therefore (\alpha+\beta+2\eta+2\delta+\gamma) < 1$

We have $p\,(f_{2n+1}, f_{2n})(x) \leq q\,p(f_{2n-1}, f_{2n})(x)$

$$\vdots$$

$$q^{2n+1}p(f_0, f_1)(x)$$

Now $p\,(f_{2n+1}, f_{2n})(x) = \sum_{i=0}^{k} p(f_{n+i-1}, f_{n+i})(x)\, Y^X$

$$\leq \frac{q^n}{1-q}\, p((f_0, f_1)(x)$$

$$\to 0 \text{ as } n \to \infty$$

Hence the sequence $\{f_n\}$ is a Cauchy sequence in Y^X. By the completeness of Y^X, there exist some f' in Y^Xsuch that

$\lim_{n\to\infty} f_n = z$

Further the continuity of T_1 implies,

$T_1(z) = T(\lim_{n\to\infty} f_n)$

$= \lim_{n\to\infty} T(f_n)$

$= z$

Thus z is a fixed point of T_1.

Similarly z is a fixed point of T_2.

Theorem 3.3: Let E ,F and T are three continuous mappings onto a housdroff sequentially complete quasi-gauge function space generated by the family p of pseudo-metric and by

ET = TE , FT = TE and $E\,(Y^X) \subset T(Y^X)$, $F\,(Y^X) \subset T(Y^X)$ (3.3.1)

$$p(Ef, Fg)(x) \leq \alpha\left[\frac{p(Tf,Ef)(x)\,p(Tg,Fg)(x)+p(Tf,Fg)(x).p(Tg,Ef)(x)}{p(Tf,Tg)(x)}\right]$$

$$+\beta\left[\frac{p(Tf,Ef)(x).p(Tf,Fg)(x)+p(Tg,Fg)(x).p(Tg,Ef)(x)}{p(Tf,Tg)(x)}\right]+\gamma\,[p(Tf,Ef)(x)+p(Tg,Fg)(x)]$$

$$+\delta[\,p(Tf,Fg)(x)+p(Tg,Ef)(x)]\;+\;\eta\,[p(Tf,Tg)(x)] \quad (3.3.2)$$

for all $f, g \in Y^X$ where $\alpha, \beta, \eta, \delta, \gamma > 0$ and $(\alpha+\beta+2\eta+2\delta+\gamma) < 1$. Then E ,F and T have

a fixed point in Y^X.

Proof: Let $f_0 \in Y^X$ and $E\,(Y^X) \subset T(Y^X)$ and $F\,(Y^X) \subset T(Y^X)$

We define a sequence $\{Tf_n\}$ as follows (3.1.1)

$Tf_{2n+1}(x) = Ef_{2n}(x)$

$Tf_{2n}(x) = Ff_{2n-1}(x)$, $\forall\, n = 1,2,3, \ldots\ldots\ldots\ldots$ and for all x in X.

Now by hypothesis of the Theorem, we have

$p\,(Tf_{2n+1}, Tf_{2n})\,(x) = p\,(Ef_{2n}(x), Ff_{2n-1})\,(x)$

$$p\;(Tf_{2n+1}, Tf_{2n})\;(x) \leq \alpha\,[\frac{p^3(Tf_{2n}, Ef_{2n})(x)+p^3(Tf_{2n}, Ff_{2n-1})(x)}{1+p^2(Tf_{2n}, Ef_{2n})(x)}]$$

$$+\beta\,[p\,(Tf_{2n-1}, Ff_{2n-1})\,(x)\,.\,p\;(Tf_{2n-1}, Ef_{2n})\;(x)] \quad +\gamma\,[p(Tf_{2n}, Tf_{2n-1})\,(x)] \quad + [p\,(Tf_{2n}, Ef_{2n})(x) + (Tf_{2n-1}, Ff_{2n-1})(x)] +$$

$$\delta\,[p\,(Tf_{2n}, Ff_{2n-1})(x) + p\,(Tf_{2n-1}, Ef_{2n})(x)\,] \quad (3.3.3)$$

$$\leq \alpha\left[\frac{\{p(Tf_{2n}, Tf_{2n+1})(x) + p(Tf_{2n}, Tf_{2n})(x)\}\{p^2(Tf_{2n}, Tf_{2n+1})(x) +}{1+p^2(Tf_{2n},}\right.$$

$$+\beta\,[p(Tf_{2n}, Tf_{2n+1})(x)]+\gamma[\,p(Tf_{2n}, Tf_{2n-1}(x)] + \eta\,[p\,(Tf_{2n}, Tf_{2n+1})(x) + p\,(Tf_{2n-1}, Tf_{2n})(x)]$$

$$+\delta\,[\,p\,(Tf_{2n-1}, Tf_{2n+1})(x)]$$

$$\leq \alpha\left[\frac{\{p(Tf_{2n}, Tf_{2n+1})(x)\}\{p^2(Tf_{2n}, Tf_{2n+1})(x)\}}{p^2(Tf_{2n}, Tf_{2n+1})(x)}\right] + \beta\,[p(Tf_{2n}, Tf_{2n+1})(x)] + \gamma[(p(Tf_{2n}, Tf_{2n-1})(x)] +$$

$\eta\,[p\,(Tf_{2n},Tf_{2n+1})(x) + p\,(Tf_{2n-1},Tf_{2n})(x)] +$
$\delta\,[\,p\,(Tf_{2n-1},Tf_{2n})(x) + p\,(Tf_{2n},Tf_{2n+1})(x)\,]$

$\leq \alpha\,[\,p\,(Tf_{2n},Tf_{2n+1})(x)] + \beta\,[p(Tf_{2n},Tf_{2n+1})(x)]$
$+\gamma[(p(\,Tf_{2n},Tf_{2n-1}\,)(x)]$

$+\eta[p(Tf_{2n},Tf_{2n+1})(x) + p(Tf_{2n-1},Tf_{2n})(x)]$
$+\delta[p(Tf_{2n-1},Tf_{2n})(x) + p(Tf_{2n},Tf_{2n+1})(x)]$

$p\,(Tf_{2n+1},Tf_{2n})(x) \leq (\alpha+\beta+\eta+\delta)\,p(Tf_{2n+1},Tf_{2n})(x) +$
$(\gamma+\eta+\delta)p\,(Tf_{2n},f_{2n-1})(x)$

$[1-(\alpha+\beta+\eta+\delta)]\,p\,(Tf_{2n+1},Tf_{2n})(x) \leq (\gamma+\eta+$
$\delta)p\,(Tf_{2n-1},Tf_{2n})(x)$

$$p\,(Tf_{2n+1},Tf_{2n})(x) \leq \frac{(\gamma+\eta+\delta)}{1-(\alpha+\beta+\eta+\delta)}\,p\,(Tf_{2n-1},Tf_{2n})(x)$$

$$\frac{(\gamma+\eta+\delta)}{1-(\alpha+\beta+\eta+\delta)} = q < 1$$

$1-(\alpha+\beta+\eta+\delta)] < (\gamma+\eta+\delta)$

$(\alpha+\beta+2\eta+2\delta+\gamma) < 1$

We have $p\,(Tf_{2n+1},Tf_{2n})(x) \leq q\,p(Tf_{2n-1},Tf_{2n})(x)$

$$\vdots$$

$$q^{2n+1}p(Tf_0,Tf_1)(x)$$

Now $p\,(Tf_{n+k},Tf_{2n})(x) = \sum_{i=0}^{k} p\,(Tf_{n+i-1},Tf_{n+i})(x)$

$$< \frac{q^n}{1-q}\,p((Tf_0,Tf_1)(x)$$

$$\to 0 \text{ as } n \to \infty$$

Hence the sequence $\{f_n\}$ is a Cauchy sequence in Y^X. By the completeness of Y^X, $\{f_n\}$ converges to a point z in Y^X.

Conclusion:

In this paper we investigations the existence of fixed point in quasi metric space with completeness.

References:

[1] P.Hitzler, Generalized metrics and topology in logic programming semantics, Ph. D Thesis, School of Mathematics, Applied Mathematics and Statistics, National University Ireland, University College Cork,(2001).

[2] S.G. Mathews, Metric domains for completeness, Technical report 76Department of Computer Science, University of Warwick, U.K, Ph.D Thesis (1985).

[3] B.E.Rhoades,A comparison of various definitions of contractive mappings, Trans of the Amer. Math. Society Vol .226(1977) P.P.257-290.

[4] Banach, sur les operations dans les ensembles abstraitsetleur application aux equations integrals, fundamental, Mathematics 3(1922),133-181.

[5] R .Kannan, Some results on fixed points, Bull .CalcuttaMath.Soc,60(1968),71-76,Math. Moan.,(1969),76,405-406 MR41,2487.

[6] A. Branciari, fixed point theorems for mappings satisfying a general contractive conditionof Integraltype,Int.J.Math.Sci,29(2002),no.531-536.

[7] I. Ekeland, Nonconvex minimization problems, Bull. Amer. Math. Soc. (N.S.)1 (3) (1979) 443–474.

[8] E. Alemany, S. Romaguera, On right K -sequentially complete quasi-metric spaces, ActaMath. Hungar. 75 (3) (1997) 267–278.

[9] A. Kristály, V.D. Radulescu, Cs. Varga, Variational Principles in Mathematical Physics, Geometry, and Economics – Qualitative Analysis of Nonlinear ˘ Equations and Unilateral

[10] Problems, Encyclopedia Math. Appl. Cambridge University Press, Cambridge, (2010)10 F. Sullivan, A characterization of complete metric spaces, Proc. Amer.Math.Soc.83 (2) (1981) 345–346.

[11] W.A. Kirk, Caristi's fixed point theorem and metric convexity, Colloq. Math. 36 (1) (1976) 81–86.

[12] S. Romaguera, A Kirk type characterization of completeness for partial metric spaces, Fixed Point Theory Appl. (2010) Art. ID 493298, 6 pp.

[13] D. Azé, J.-N. Corvellec, On some variational properties of metric spaces, J. Fixed Point Theory Appl. 5 (1) (2009) 185–200.

[14] H. Brézis, F.E. Browder, A general principle on ordered sets in nonlinear functional analysis, Advances in Math. 21 (3) (1976) 355–364.

[15] J. Caristi, Fixed point theorems for mappings satisfying inwardness conditions, Trans.Amer. Math. Soc. 215 (1976) 241–251.

[16] J. Caristi, W.A. Kirk, Geometric fixed point theory and inwardness conditions, in: TheGeometry of Metric and Linear Spaces, Proc. Conf., Michigan State Univ., East Lansing, Mich., (1974) in Lecture Notes in Math., vol. 490, Springer, Berlin,(1975) pp. 74–83.

[17] S.A. Chen, W. Li, S.P. Tian, Z.Y. Mao, On optimization problems in quasi-metric spaces, In Proc. 5th

International Conf. on Machine Learning and Cybernetics, Dalian, 13–16 ,Aug. (2006) pp. 865–870.

[18] S. Chen, S. Tian, Z. Mao, On Caristi's fixed point theorem in quasi-metric spaces, Dyn.Contin. Discrete Impuls. Syst. Ser. A Math. Anal. 13A (Part 3, suppl.) (2006) 1150–1157

[19] S.A. Chen, W. Li, D. Zou, S.B. Chen, Fixed point theorems in quasi-metric spaces, in: Proc. 6th International Conf. on Machine Learning and Cybernetics, Hong Kong, 19–22 Aug. (2007) pp. 2499–2504.

[20] L.Ciri ´ c, Periodic and fixed point theorems in a quasi-metric space, J. Austral. Math. Soc. Ser. A 54 (1) (1993) 80–85.

[21] S. Cobzas¸ Asymmetric locally convex spaces, Int. J. Math. Math. Sci. (16) (2005) 2585–2608.

[22] S. Cobza¸s, Compact and precompact sets in asymmetric locally convex spaces, Topology Appl.156 (9) (2009) 1620–1629

[23] S.Cobza¸s, Functional analysis inasymmetric normed spaces, arXiv:1006.1175v1, (2010).

[24] I. Ekeland, On the variational principle, J. Math. Anal. Appl. 47 (1974) 324–353.

[25] P. Fletcher, W.F. Lindgren, Quasi-Uniform Spaces, M. Dekker, New York, (1982).

[26] L.M. García-Raffi, S. Romaguera, E.A. Sánchez-Pérez, On Hausdorff asymmetric normed linear spaces, Houston J. Math. 29 (3) (2003) 717–728.

[27] L.M. García-Raffi, S. Romaguera, E.A. Sánchez-Pérez, Weak topologies on asymmetric normed linear spaces and non-asymptotic criteria in the theory of complexity analysis of algorithms, J. Anal. Appl. 2 (3) (2004) 125–138.

[28] T.L. Hicks, Fixed point theorems for quasi-metric spaces, Math. Japon. 33 (1988) 231–236.

[29] J.C. Kelly, Bitopological spaces, Proc. London Math. Soc. 13 (1963) 71–89.

[30] H.-P.A. Künzi, M. Mrševic, I.L. Reilly, M.K. Vamanamurthy, Convergence, precompactness and symmetry in quasi-uniform spaces, Math. Japon. 38 (1993)´ 239–253.

[31] A.C.G. Mennucci, On asymmetric distances, preprint, 2nd version 2007 (1st version 2004) bothavailable at http://cvgmt.sns.it/people/mennucci.

[32] I.L. Reilly, P.V. Subrahmanyam, M.K. Vamanamurthy, Cauchy sequences in quasi-pseudo-metric spaces, Monatsh. Math. 93 (1982) 127–140.

[33] S. Romaguera, Left K-completeness in quasi-metric spaces, Math. Nachr. 157 (1992) 15–23.

[34] S. Romaguera, A. Gutiérrez, A note on Cauchy sequences in quasipseudometric spaces, Glas. Mat.Ser. III 21(41) (1) (1986) 191–200.

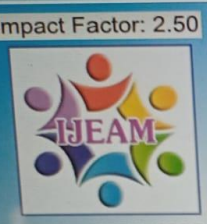
[35] S. Romaguera, M. Schell kens, Quasi-metric properties of complexity spaces, Topology

[36] S. Romaguera, M. Schell kens, Duality and quasi-normability for complexity spaces, Appl. Gen .Topol. 3 (1) (2002) 91–112.

[37] R.A. Stoltenberg, On quasi-metric spaces, Duke Math. J. 36 (1969) 65–71.

[38] J. Sh. Ume, A minimization theorem in quasi-metric spaces and its applications, Int. J. Math. Math. Sci. 31 (7) (2000).

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Study of Fixed Point Theorems with b-Metric Space

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Abstract: In this paper we have access some fixed point theorems on b-metric space which is an more extension of a some fixed point theorems by Kannan [2] and Chatterjea [3]. Also we known that any contraction mapping is not necessary to be a weak contraction mapping in b-metric spaces.
Keyword: b-metric space , fixed point ,contraction mappings ,cauchy sequence ,complete metric space.
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I. INTRODUCTION AND PRELIMINARIES

We know that fixed point theory is most important topic in development of mathematics in nonlinear analysis and other branches of mathemetics. Since fixed point theory has been used capable in many other branch of science, chemistry, biology, economics, computer science, engineering, and etc. It is known that Banach contraction mapping is a good example. However metric fixed point theory is vast field of study and is effectively of solving many fixed point equations. To overcome the problem of measurable functions w.r.t. a measure and their convergence,Czerwik [4] needs an extension of metric space. Using this idea, he presented a generalization of the renowned Banach fixed point theorem in the b-metric spaces (see also [9,10,11]). Many researchers including Aydi [5], Boriceanu [6,22,23], Bota [7], Chug [8], Du [12],Kir [13], Olaru [14], Olatinwo [15], Păcurar [16,17], Rao [18],Roshan [20] and Shi [21] studied the extension of fixed point theorems in b-metric space.. In this paper, our aim is to show the validity of some important fixed point results into b-metric spaces and solve by different methods .We recall some definitions and properties for b-metric spaces and fixed point given by Czerwik [4]. Some problems, particularly the problem of the convergence of measurable functions with respect to a measure, lead to a generalization of notation of metric.Using this idea, Czerwik in [4] and [5] presented a generalization of the well known Banach's fixed point theorem in so called b-metric spaces. Our purpose is to show that some well-known fixed point theorems are valid in b-metric spaces. It is interesting that we see some properties in (usual) metric space is not valid in b-metric space.

In this section we consider the basic definitions and properties of b-metric space and unique fixed point. we give some basic definitions and results on b-metric space.

We recall some definitions and properties for b-metric spaces given by Czerwik [4]

Definition 2.1: [4]Let (M, ρ) be a b-metric space then x_n in M is called
A Cauchy sequence iff $\forall\ \epsilon > 0$ there exists $n(\epsilon) < \square$ such that for each n, $m \geq n\ \epsilon$
We have $\rho\ x_n , x_m < \square$.
(b) Convergent sequence if and only if there exist $x \in M$ such that for all $\epsilon > 0$ there exists $n(\epsilon) < \square$,such that for every $n \geq n\ \epsilon$ we have $\rho\ x_n , x < \square$.

Definition 2.2: [4] If (M , ρ) is a b-metric space then a subset $L \square M$ is called
Compact iff for every sequence of elements of L there exists a subsequence that converges to an element of L.
Closed iff for each sequence x_n in L which converges to an element x, we have $x \square L$.
(c) The b-metric space is complete if every Cauchy sequence converges.

Definition 2.3: [8] Let X be a nonempty set and let $s \geq 1$ be a given real number. A function $d \square X \times X \rightarrow R^+$ is called a b-metric provided that, for $x, y, z \in X$
(a) $d(x ,y) = 0$ if and only if $x = y$
(b) $d(x, y) = d(y ,x)$,
(c) $d\ x, z \leq s[\ d\ x , y\ +\ d\ (y , z)]$. (2.1)

A pair (X, d) is called a b-metric space. It is clear that the definiton of b-metric space is a extension of usual metric space. Also, if we consider $s = 1$ in Definition 2.1(c), then we obtain definition of usual metric space. By this reason,

our results are more general than the same results in (usual) metric space. We can give some example of b-metric space as following:

Example 2.1: The space l_p ,($0 < p < 1$)
$l_p = x_n \subset R \sum_{n=1}^{\infty} x_n^p < \infty$,
together with the function $\rho : l_p \times l_p \to R$ where

$$\rho(x, y) = \sum_{n=1}^{\infty} |x_n - x_y|^p \quad \frac{1}{p}$$

Where $x = x_n$, $y = y_n \in l_p$ is a b -metric space. By an elementary calculation we obtain that

$$\rho(x, y) = 2^p [\rho(x, y) + \rho(y, z)]$$

Example 2.2: Let X= [1,∞).Define a function $d : X \times X \to [1, \infty)$ such that $d(x, y) = |x - y|^2$ for all $x, y \in X$. Then (X ,d) is a complete b-metric space on X with constant s = 2.Also d is not a metric on X.

Definition 2.4: [6] Let (X, d) be a b-metric space. Then a sequence x_n in X is called Cauchy sequence if and only if for all $\varepsilon > 0$ there exists $n(\varepsilon) \in N$ such that for each n, $m \geq n$ ε we have $\rho(x_n, x_m) < \varepsilon$.

Definition 2.5: [6] Let (X, d) be a b-metric space. Then a sequence x_n in X is called convergent sequence if and only if there exists $x \in X$ such that for all $n(\varepsilon) \in N$, $n \geq n$ ε ,we have $d(x_n, x) < \varepsilon$. In this case we write $\lim_{n\to\infty} x_n = x$.

Definition 2.6: [6] The b-metric space is complete if every Cauchy sequence convergent.

Definition 2.7: [22] Let E be a nonempty set and $T : E \to E$ a self map. We say that $x \in X$ is a fixed point of T if T (x) = x and denote by FT or Fix (T) the set of all fixed points of T. Let E be any set and $T : E \to E$ a self map. For any given $x \in X$,we define $T^n(x)$ inductively by $T^n(x) = x$ and $T^{n+1} x = T\, T^n(x)$; we recall $T_n(x)$ the nth iterative of x under T.

For any $x_0 \in X$,the sequence $\{x_n\}_{n\geq 0} \subset X$ given by

$$x_n = T_{x_{n-1}} = T^n_{x_0}, \quad n = 1,2,3\ldots.. \tag{2.2}$$

(2.2) is called the sequence of successive approximations with the initial value x_0 . It is also known as the picard iteration starting at x_0.

Definition 2.8: [4] Let (X, d) be a metric space. A mapping $T : X \to X$ is called weak contraction if there exists a constant $\delta \in [0,1)$ and some $L \geq 0$ such that

$$d(T_x, T_y) \leq \delta\, d(x, y) + L\, (y, T_x) \tag{2.3}$$

for all $x, y \in X$

Remark 1: [4] Due to the symmetry of the distance, the weak contractive condition (2.2) implicitly includes the following dual one

$$d(T_x, T_y) \leq \delta\, d(x, y) + L\, (y, T_y) \tag{2.4}$$

for all $x, y \in X$,Consequently, in order to check the weak contractiveness of T; it is necessary to check both (2.3) and (2.4).

Definition 2.9: Let (X ,d) be a b-metric space and $T : X \to X$ be a given mapping .We say that T is α - admissible if $x, y \in X$, $\alpha(x, y) \geq 1$ implies that $\alpha(T_x, T_y) \geq 1$.

Example 2.3: Let $X = (0, \infty)$ .Define $T : X \to X$ and $\alpha: X \times X \to [0, \infty)$ by $T_x = \ln x$

For all $x, \in X$ and

$$\alpha(x, y) = \begin{cases} 2 & \text{if } x \geq y \\ 0 & \text{if } x \geq y \end{cases}$$

Then T is α – admissible.

III. MAIN RESULT

Let (X, d) be a complete b-metric space with constant $s \geq 1$ and define the sequence $\{x_n\}_{n=1}^{\infty} \subset k$ by the recursion 2.1(c). Let $T : X \to X$ be a mapping for which there exists $\mu \in [0, \frac{1}{2})$ such that

$$d(Tx, Ty) \leq \mu\, d(x, Tx) + d(y, Ty) \qquad (A)$$

for all $x, y \in X$.
Then there exists $x^{*} \in X$ such that $x_n \to x^{*}$ and x^{*} is unique fixed point of T.

Theorem 3.1: Let (X ,d) be a complete b-mertic space with constant $s \geq 1$ and define the sequence $\{x_n\}_{n=1}^{\infty} \subset K$ by the recursion 2.3 (c),
Let $T: X \to X$ be a mapping for which there exists, $\mu \in [0, \frac{1}{2})$, Such that

$$d(Tx, Ty) \leq \mu\, d(x, Tx) + d(y, Ty) + \delta\, d(x, Ty) + d(y, Tx) + \eta\, d(x, y) + \beta \frac{d(x, Ty) + d(y, Tx)}{1 + d(x, Tx)\, d(y, Ty)\, d(x, Ty)\, d(y, Tx)} \qquad (3.1.1)$$

for all $x, y \in X$ and $\mu, \delta, \eta, \beta \in X$.
Then there exists $x' \in X$ such that $x_n \to x'$ and x' is unique fixed point of T.
Proof: Let $x_0 \in X$ and $\{x_n\}_{n=1}^{\infty}$ be a sequence in X defined as $x_n = Tx_{n-1} = T^{n}_{x_0}$, $n = 1,2,3,4 \ldots\ldots\ldots$ by using (A) and (2.3) and (2.7) . We know that

$$d(x_n, x_{n+1}) = d(Tx_{n-1}, Tx_n)$$

$$d(x_n, x_{n+1}) \leq \mu\, d(x_{n-1}, Tx_{n-1}) + d(x_n, Tx_n) + \delta\, d(x_{n-1}, Tx_n) + d(x_n, Tx_{n-1}) + \eta\, d(x_{n-1}, x_n) + \beta \frac{d(x_{n-1}, Tx_n) + d(x_n, Tx_{n-1})}{1 + d(x_{n-1}, Tx_{n-1})\, d(x_n, Tx_n)\, d(x_{n-1}, Tx_n)\, d(x_n, Tx_{n-1})}$$

$$\leq \mu\, d(x_{n-1}, x_n) + d(x_n, x_{n+1}) + \delta\, d(x_{n-1}, x_{n+1}) + d(x_n, x_n) + \eta\, d(x_{n-1}, x_n) + \beta \frac{d(x_{n-1}, x_{n+1}) + d(x_n, x_n)}{1 + d(x_{n-1}, x_n)\, d(x_n, x_{n+1})\, d(x_{n-1}, x_{n+1})\, d(x_n, x_{n+1})}$$

$$\leq \mu\, d(x_{n-1}, x_n) + d(x_n, x_{n+1}) + \delta\, d(x_{n-1}, x_{n+1}) + \eta\, d(x_{n-1}, x_n) + \beta\, d(x_{n-1}, x_{n+1})$$

$$d(x_n, x_{n+1}) \leq \mu\, d(x_{n-1}, x_n) + d(x_n, x_{n+1}) + \delta\, d(x_{n-1}, x_n) + d(x_n, x_{n+1}) + \eta\, d(x_{n-1}, x_n) + \beta [d(x_{n-1}, x_n) + d(x_n, x_{n+1})]$$

$$d(x_n, x_{n+1}) \leq (\mu + \delta + \eta + \beta)\, d(x_{n-1}, x_n) + (\mu + \delta + \beta)\, d(x_n, x_{n+1}) \qquad (3.1.2)$$

$$1 - (\mu + \delta + \beta)\, d(x_n, x_{n+1}) \leq (\mu + \delta + \eta + \beta)\, d(x_{n-1}, x_n)$$

$$d(x_n, x_{n+1}) \leq \frac{\mu + \delta + \eta + \beta}{1 - (\mu + \delta + \beta)} d(x_{n-1}, x_n)$$

Similarly $d(x_n, x_{n+1}) \leq \left(\frac{\mu + \delta + \eta + \beta}{1 - (\mu + \delta + \beta)}\right)^{n} d(x_0, x_1)$

Note that $\mu \in [0, \frac{1}{2})$ Then $\frac{\mu + \delta + \eta + \beta}{1 - (\mu + \delta + \beta)} \in [0, \frac{1}{2})$.
Thus T is a contraction mapping. We know that $\{x_n\}_{n=1}^{\infty}$ is a Cauchy sequence and hence a convergent sequence ,We consider that $\{x_n\}_{n=1}^{\infty}$ convergent to $x' \in X$,
Then we have

$$d(x', Tx') \leq s[d(x', x_{n+1}) + d(x_{n+1}, x_n) + d(x_n, Tx')]$$

$$\leq s[d(x', x_{n+1}) + d(x_{n+1}, x_n) + d(Tx_{n-1}, Tx')]$$

$$\leq s\left[d(x', x_{n+1}) + d(x_{n+1}, x_n) + \mu\, d(x_{n-1}, Tx_{n-1}) + d(x', Tx') + \delta\, d(x_{n-1}, Tx') + d(x', Tx_{n-1}) + \eta\, d(x_{n-1}, x') + \beta \frac{d(x_{n-1}, Tx') + d(x', Tx_{n-1})}{1 + d(x_{n-1}, Tx_{n-1})\, d(x', Tx')\, d(x_{n-1}, Tx')\, d(x', Tx_{n-1})}\right]$$

$$\leq s \begin{matrix} d\,x', x_{n+1} + d\,x_{n+1}, x_n + \mu[d\,x_{n-1}, x_n + d\,x', T_{x'}] \\ +\delta\, d\,x_{n-1}, T_{x'} + d\,x', x_n + \eta\, d\,x_{n-1}, x \end{matrix} + \beta \frac{d\,x_{n-1}, T_{x'} + d\,x', x_n}{1 + d\,x_{n-1}, x_n\, d\,(x', T_{x'})\, x_{n-1}, T_{x'}\, d\,x', x_n}$$

$$(1 - s\mu)\, d(x', T_{x'}) \leq s \begin{matrix} d\,x', x_{n+1} + d\,x_{n+1}, x_n + \mu d\,x_{n-1}, x_n \\ +\delta\, d\,x_{n-1}, T_{x'} + d\,x', x_n + \eta d\,x_{n-1}, x \end{matrix} + \beta \frac{d\,x_{n-1}, T_{x'} + d\,x', x_n}{1 + d\,x_{n-1}, x_n\, d\,(x', T_{x'}) d\,x_{n-1}, T_{x'}\, d\,x', x_n}$$

$$d(x', T_{x'}) \leq \frac{s}{(1 - s\mu)} \begin{matrix} d\,x', x_{n+1} + d\,x_{n+1}, x_n + \mu d\,x_{n-1}, x_n \\ +\delta\, d\,x_{n-1}, T_{x'} + d\,x', x_n + \eta d\,x_{n-1}, x \end{matrix} + \beta \frac{d\,x_{n-1}, T_{x'} + d\,x', x_n}{1 + d\,x_{n-1}, x_n\, d\,(x', T_{x'}) d\,x_{n-1}, T_{x'}\, d\,x', x_n}$$

Lim $n \to \infty$, $\lim_{n\to\infty} d\,x', T_{x'} = 0$

$\therefore x' = T_{x'}$

x' is the fixed point of T.

Theorem 3.2: let (X ,d) be a complete b-metric space and define the sequence $\{x_n\}_{n=1}^{\infty} \subset K$ by the recursion(2.3) and (2.7).

Let $T: X \to X$ be a mapping under the terms, $s\lambda \in [0, \frac{1}{2})$, Such that

$$d(T_x, T_y) \leq \lambda [\, d(x, T_y) + d(y, T_x)\,] \qquad (3.2.1)$$

for all $x, y \in X$. Then there exist $x \in X$ such that $x_n \to x'$ and x' is unique fixed point of T.

Proof : Let $x_0 \in X$, $x_n = T_{x_{n-1}} = T^n_{x_0}$, n-1,2,3 ---------

$$d(x_{n+1}, x_{n+2}) = d(T_{x_n}, T_{x_{n+1}})$$
$$\leq \lambda [\, d(x_n, T_{x_{n+1}}) + d(x_{n+1}, T_{x_n})\,]$$
$$\leq \lambda [\, d(x_n, x_{n+2}) + d(x_{n+1}, x_{n+1})\,]$$
$$\leq \lambda [d(x_n, x_{n+2})]$$
$$d(x_{n+1}, x_{n+2}) \leq s\lambda [\, d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+2})\,] \quad (\text{by } 2.3)$$
$$(1 - s\lambda)\, d(x_{n+1}, x_{n+2}) \leq s\lambda [\, d(x_n, x_{n+1})]$$
$$d(x_{n+1}, x_{n+2}) \leq \frac{s\lambda}{(1 - s\lambda)} d(x_n, x_{n+1}) \qquad (3.2.2)$$

Note that $\frac{s\lambda}{(1 - s\lambda)} \in [0, \frac{1}{2})$.

Thus T is contraction mapping, by using (A) and Theorem-3.1, We see that $\{x_n\}_{n=1}^{\infty}$ is a cauchy sequence then a convergent sequence .

Now we show that x' is a fixed point of T.

$$d(x', T_{x'}) \leq s[d\,x', x_n + d(x_n, T_{x'})]$$
$$\leq s[d\,x', x_n + d(T_{x_{n-1}}, T_{x'})]$$
$$\leq s \begin{matrix} d\,x', x_n + \mu d\,x_{n-1}, T_{x_{n-1}} + d\,x', T_{x'} \\ +\delta\, d\,x_{n-1}, T_{x'} + d\,x', T_{x_{n-1}} + \eta\, d\,x_{n-1}, x' \end{matrix} + \beta \frac{d\,x_{n-1}, T_{x'} + d\,x', T_{x_{n-1}}}{1 + d\,x_{n-1}, T_{x_{n-1}}\; d\,x', T_{x'}\, d\,x_{n-1}, T_{x'}\, d\,x', T_{x_{n-1}}}$$
$$\leq s \begin{matrix} d\,x', x_n + \mu d\,x_{n-1}, x_n + d\,x', T_{x'} \\ +\delta\, d\,x_{n-1}, T_{x'} + d\,x', x_n + \eta\, d\,x_{n-1}, x' \end{matrix} + \beta \frac{d\,x_{n-1}, T_{x'} + d\,x', x_n}{1 + d\,x_{n-1}, x_n\, d(x', T_{x'}) d\,x_{n-1}, T_{x'}\, d\,x', x_n}$$

Lim $n \to \infty$, $d(x', T_{x'}) \leq s\mu\, d(x', T_{x'})$ (3.2.3)

(3.2.3) is false unless $d(x', T_{x'}) = 0$

Thus we arrive at $x' = T_{x'}$.
Now we show that x' is the unique fixed point of T.We assume that x' is another fixed point of T. Then we have x = T(x) , y = T(y)

$$d(x ,y) = \mu\, d\,x, T(x) + d\,y, T(y) + \delta\, d\,x, T(y + d\,y, T(x) + \eta\, d(x, y) + \beta \frac{d\,x,T(y), +d\,y,T(x)}{1+ d\,x,T(x)\, d\,y,T(y)\, d\,x,T(y)\, d\,y,T(x)} \quad (3.2.4)$$

$$d (x ,y) = \mu\ d\,x, x\ + d\,y, y\ + \delta\, d\,x, y\ + d\,y, x\ + \eta d(x, y) + \beta \frac{d\,x,y +d\,y,T(x)}{1+ d\,x,x)\, d\,y,y)\, d\,x,y\, d\,y,x}$$

$$d (x ,y) = \mu\, d\,x, x + d\,y, y + \delta\, d\,x, y + d\,y, x + \eta\, d(x, y) + \beta[d\,x, y + d\,y, x\ ]$$

$$d (x ,y) \leq (2\delta + \eta + 2\beta)\, d\,(x,y)$$

Which is a contraction .This implies that $x = x_0$
This complete the proof.

Theorem 3.3: let (X ,d) be a complete b-metric space with metric d and let S & □: □ → □ are two functions with the following property.

$$d\,(\,S_x\,,\,T_y\,) \leq \mu\, d(x\,,\,S_x\,) + d\,y,\,T_y + \delta\, d\,x,\,S_x + d\,y,\,T_y\,) + \eta\, d(\,x,\,y)$$
$$+ \beta \frac{d\,x,,T_y +d\,y,S_x\,)}{1+ d\,x\,,S_x\ d\,y,T_y\ d\,x,,T_y\ \ d\,y,S_x}$$

for all x , y ϵ X , s ≥ 1 . Then S & T has a unique fixed point.
Proof : Let x_0 ϵ X and x_n be a sequence in X. Such that
$x_1 = S(x_0)$, $x_2 = T\,(x_1)$
$x_{2n} = T\,(x_{2n-1})$, $x_{2n+1} = S(x_{2n})$
Now $d(x_{2n+1},x_{2n+2}) = d(\,Sx_{2n,}Tx_{2n+1})$

$$d(x_{2n+1}\,,\,x_{2n+2}) \leq \mu\, d(\,x_{2n}\,,\,Sx_{2n}\,) + d\,x_{2n+1}\,,T\,x_{2n+1} + \delta\, d\,x_{2n+1}\,,\,Sx_{2n} + d\,x_{2n}\,,\,T\,x_{2n+1}$$
$$+ \eta d\,x_{2n}\ \ ,x_{2n+1}\ \ + \beta \frac{d\ \ x_{2n}\,,T\,x_{2n+1}\,+d(x_{2n+1}\,,Sx_{2n}\,)}{1+d(\,x_{2n}\,,Sx_{2n,})d\,x_{2n+1}\,,T\,x_{2n+1}\ d\ \ x_{2n}\,,T\,x_{2n+1}\ d(x_{2n+1}\,,Sx_{2n}\,)}$$
$$\leq \mu\, d(\,x_{2n}\,,\,x_{2n+1}\,) + d\,x_{2n+1}\,,\,x_{2n+2}\ \ + \eta\ d\ \ x_{2n}\,,\,x_{2n+1}\ \ + \delta\, d\,x_{2n+1}\ \ ,\,x_{2n+1}\ \ +\ \ \ \ \ \ \ \ \ \ d\,x_{2n}\,,\,x_{2n+2}\ +$$
$$\beta \frac{d\ \ x_{2n}\,,x_{2n+2}\ \ +d(x_{2n+1}\,,x_{2n+1}\,)}{1+d(\,x_{2n}\,,x_{2n+1}\,)d\,x_{2n+1}\,,\,x_{2n+2}\ d\ \ x_{2n}\,,x_{2n+2}\ d(x_{2n+1},x_{2n+1})}$$
$$\leq \mu\, d\,x_{2n}\,,\,x_{2n+1} + d\,x_{2n+1}\,,\,x_{2n+2} + \eta d\,x_{2n}\,,\,x_{2n+1}$$
$$+ \delta\, d\,x_{2n+1}\,,\,x_{2n+1} + d\,x_{2n}\,,\,x_{2n+2}$$
$$\leq \mu\, d\,x_{2n}\,,\,x_{2n+1} + d\,x_{2n+1}\,,\,x_{2n+2} + \eta d\,x_{2n}\,,\,x_{2n+1} + \delta d\,x_{2n}\,,\,x_{2n+2}$$
$$\leq \mu\, d\,x_{2n}\,,\,x_{2n+1} + d\,x_{2n+1}\,,\,x_{2n+2} + \eta d\,x_{2n}\,,\,x_{2n+1}$$
$$+ \delta[d\ \ x_{2n}\,,\,x_{2n+1}\ + d\ \ x_{2n+1}\,,\,x_{2n+2}\ ] \quad (3.3.1)$$

If $d(x_{2n+1}\,,\,x_{2n+2}) \geq d\,x_{2n}\,,\,x_{2n+1}$ Then we have

$$d(x_{2n+1}\,,\,x_{2n+2}\,) \leq \mu\, d\,x_{2n+1}\,,\,x_{2n+2} + d\,x_{2n+1}\,,\,x_{2n+2} + \eta d\,x_{2n+1}\,,\,x_{2n+2}$$
$$+ \delta\ s[d\,x_{2n+1},\,x_{2n+2} + d\,x_{2n+1}\,,\,x_{2n+2}\ ]$$
$$d(x_{2n+1}\,,\,x_{2n+2}) \leq (\,2\mu + \eta +2\delta\,)\, d\ \ x_{2n+1}\,,x_{2n+2} \quad (3.3.2)$$

This is a contraction.

From the above inequality ,we deduce that

$$d(x_{2n+1}\,,x_{2n+2}) \leq (\,2\mu + \eta +2\delta\,)\, d\,x_n\,,\,x_{n+1}$$

Continuing this process we can in general

$$d\ \ x_n\,,\,x_{n+1}\ \ \leq K^n\, d\ \ x_0\,,x_1 \quad (3.3.3)$$

This implies that T is a contraction mapping.
Now it is show that $\{x_n\}$ is a Cauchy sequence in X.
Let m , n > 0 with m > n

$$d\,x_n\,,\,x_m \leq s\, d\,x_n\,,\,x_{n+1} + d\,x_{n+1}\,,\,x_m$$
$$\leq sd\,x_n\,,\,x_{n+1} + s^2\, d\,x_{n+1},\,x_{n+2} + s^3\, d\,x_{n+2}\,,\,x_{n+3}\ + + + + - - - -$$
$$\leq sK^n\, d\,x_0,\,x_1 + s^2\,K^{n+1}\, d\,x_0,\,x_1 + s^3\,K^{n+3}\, d\,x_0,\,x_1 \text{ +++++----------}$$
$$\leq sK^n_{sK}\, d\,x_0,\,x_1\, 1 + sK + (sK)^2 + (sK)^3 + + + + - - - -$$
$$\leq \frac{^{\ n}}{1-sK}\ d\ \ x\ \ ,x_{0\ 1} \quad (3.3.4)$$

Taking limit $n \to \infty$, we get $\lim_{n \to \infty} d(x_n, x_m) = 0$
$\therefore$ x_n is a Cauchy sequence in X is complete.
We consider that x_n converse to u.
Now we show that u is fixed point of S & T .
We have $d(u, Su) \le s\, d(u, x_n) + d(x_n, Su)$ (3.3.5)
$d(u, Su) \le s\, d(u, x_n) + d(Tx_{n-1}, Su)$

$$\le s \left[\begin{array}{c} d(u, x_n) + \mu\, d(u, Su) + d(x_{n-1}, Tx_{n-1}) \\ + \delta\, d(u, Tx_{n-1}) + d(x_{n-1}, Su) + \eta\, d(u, x_{n-1}) \\ d(u, Tx_{n-1}) + d(x_{n-1}, Su) \end{array} \right] + \beta \frac{}{1 + d(u, Su)\, d(x_{n-1}, Tx_{n-1})\ d(u, Tx_{n-1})\ d(x_{n-1}, Su)}$$

$$d(u, Su) \le s \left[\begin{array}{c} d(u, x_n) + \mu\, d(u, Su) + d(x_{n-1}, x_n) \\ + \delta\, d(u, x_n) + d(x_{n-1}, Su) + \eta\, d(u, x_{n-1}) \\ d(u, x_n) + d(x_{n-1}, Su) \end{array} \right) + \beta \frac{}{1 + d(u, Su)\, d(x_{n-1}, x_n)\ d(u, x_n)\ d(x_{n-1}, Su)}$$

taking limit $n \to \infty$, we get
$d(u, Su) \le s\, \mu d(u, Su)$
$(1 - s\, \mu)\, d(u, Su) \le 0$
Which gives $d(u, Su) = 0 \square Su = u$
Hence u is a fixed point of S.
Similarly we can show that u is a fixed point of T.
Hence u is a common fixed point of S & T.
Uniqueness: Let u & v be two fixed point of S & T.
Such that $u \ne v$,Putting x = u & y = v in Theo.(3.1),
We have $d(u, v) = d(Su, Tv)$
$\le \mu\, d(u, Su) + d(v, Tv) + \delta\, d(u, Tv) + d(v, Su) + \eta\, d(u, v)$
$+ \beta \frac{d(u, Tv) + d(v, Su)}{1 + u, Su\, d(v, Tv\, d(u, Tv\, d(v, Su}$
$\le \mu\, d(u, u) + d(v, v) + \delta\, d(u, v) + d(v, u) + \eta\, d(u, v)$
$+ \beta \frac{d(u, v) + d(v, u)}{1 + u, u\, d(v, v\, d(u, v\, d(v, u}$
$\le (2\delta + \eta + 2\beta)\, d(u, v)$
This is a contraction.

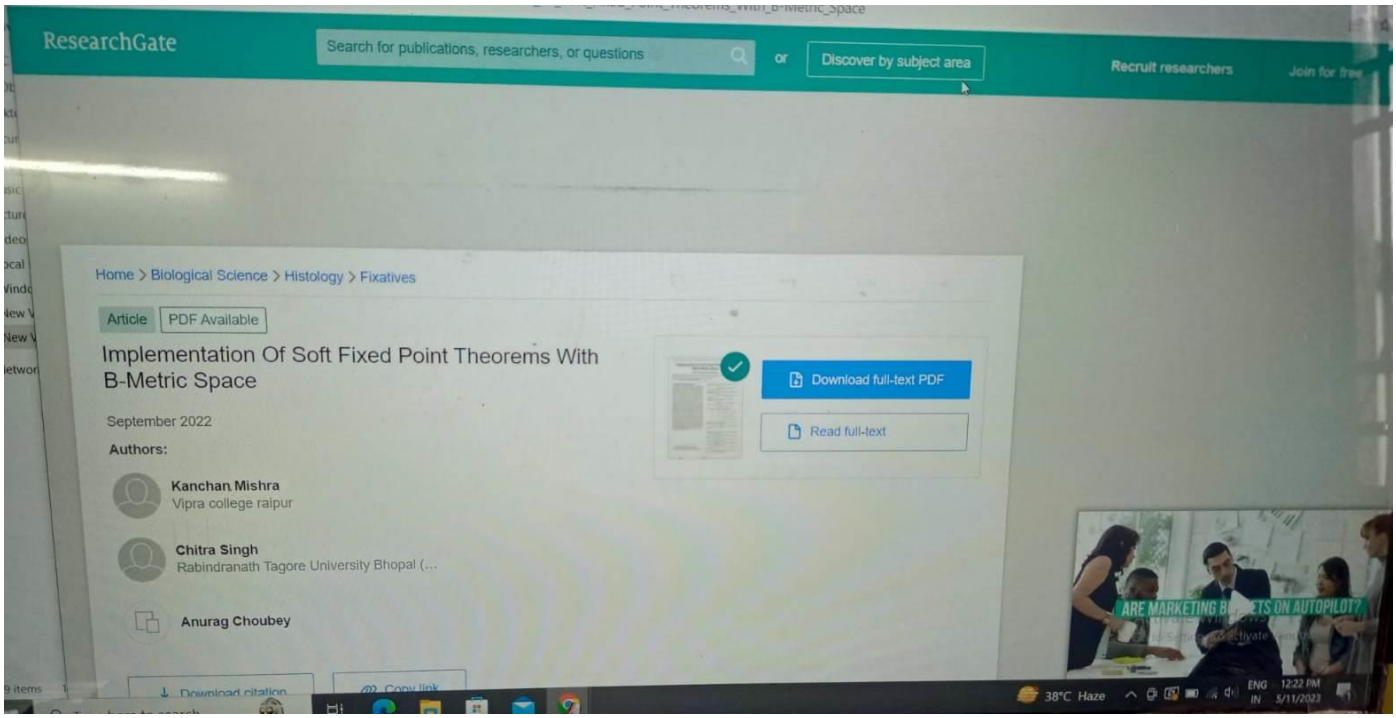
IV. CONCLUSION

In this paper we proof some unique fixed point theorems in b-metric space. We have apply different method to solve b-metric space theorems and discussed in detail the fundamentals of b-metric space theory.

V. REFERENCES:

[1] Pankaj Kumar Mishra,Shweta Sachdeva,S.K Banerjee,Some Fixed Point Theorems in bmetric Space,University of Petroleum & Energy Studies,Vol.2,No.1,19-22,(2014).
[2] Kannan , R, Some results on fixed points, Bull. Calcutta Math. Soc., 60, 71-76, (1968).
[3] Chatterjea, S. K, Fixed point theorems, C.R. Acad. Bulgare Sci. 25727-730,(1972).
[4] S. Czerwik,Contraction mappings in b-metric spaces, ActaMathematica etInformatica Universitatis Ostraviensis 15-11,(1993).
[5] Hassen Aydi, Monica-Felicia Bota, Erdal Karapinar, and Slobodanka Mitrović, A fixed point theorem for set-valued quasi contractions in b-metric spaces ,Fixed Point Theory and Application no. 88 ,(2012).
[6] Monica Boriceanu, Fixed point theory for multivalued generalized contraction on aset with two b-metrics, Babes-Bolyai University Mathematica Studia LIV, no. 3, 1-14,(2009).
[7] M . Bota, A. Molnar, and C. Varga, On ekeland's variational principle inb-metric spaces, Fixed Point Theory 12 no. 2, 21-28, (2011).
[8] Renu Chugh, Vivek kumar, and Tamanna Kadian, Somefixed point theoremsfor multivalued mappings in generalized b-metric spaces, International Journal of Mathematical Archive 3 no. 3, 1198-1210,(2012).
[9] Czerwik S.Nonlinear set-valued contraction mappings in b-metric spaces, Atti.Sem. Mat. Fis. Univ. Modena46,263-276(1998).
[10] S. Czerwik, Krzyszt Dlutek, and S. L. Singh, Round-off stability of iterationprocedures for operators in b-metric spaces, J. Natur. Phys. Sci. 11, 87-94 (1997).
[11] Czerwik S, Dlutek K, and Singh S L.Round-off stability of iteration procedures for set-valued operators in b-metric spaces, J. Natur. Phys. Sci. 15, 1-2(2001).

[12] Wei Shih Du and Erdal Karapýnar, A note on cone b-metric and its related results: generalizations or equivalence ?, Fixed Point Theory and Application no. 210 , (2012).
[13] G. E. H Mehmet Kir and Hukmi Kiziltunc, On some well known fixed point theorems in b-metric spaces,Turkish Journal of Analysis and Number Theory, 1 (2013).
[14] Ion Marian Olaru and Adrian Branga, Common fixed point results in b-k-metric spaces,General Mathematics 19 no. 4, 51-59,(2011).
[15] Memudu O. Olatinwo and Christopher O. Imoru, A generalisation of some results on multi-valued weakly Picard mappings in b-metric space ,Fasciculi - Mathematici 40 45-56 (2008).
[16] Mădălina Păcurar, A fixed point result for φ-contractions on b-metric spaceswithout the boundedness assumption,Fascicyli Mathematici 43 125-137,(2010).
[17] Pacurar M.Sequences of almost contractions and fixed points in b-metric spaces, Analele Universităţ de Vest, Timisoara, Seria MatematicăInformatică XLVIII no.3,125- 137,(2010).
[18] K. P. R. Rao and K. R. K. Rao, A common fixed point theorem for two hybridpairs of appings in b-metric spaces, International Journal of Analysis (2013).
[19] Simeon Reich,Some remarks concerning contraction mappings,CanadianMathematical Bulletins 14 (1971), no. 1,121-124.
[20] Jamal Rezaei Roshan, Vahid Parvaneh, Shaban Sedghi, Nabi Shobkolaei,and Wasno. 1, 13{16ardy and T. D. Rogers, A generalization of fixed point theorem of reich , Canadian Mathematical Bulletin 16 (1973), 201-206.
[21] Lu Shi and Shaoyuan Xu, Common fixed point theorems for two weakly com-patible self-mappings in cone b-metric spaces, Fixed Point Theory and Applications2013, no. 120.
[22] Boriceanu M.Fixed point theory on spaces with vector-valued b- metrics,DemonstratioMathematica XLII no. 4,285-301,(2009).
[23] Boriceanu M.Strict fixed point theorems for multivalued operators in b-metric spaces.International Journal of Modern Mathematics 4 (2009), no. 2, 285-301.
[24] Choudhury, B. S., Unique fixed point theorem for weakly c-contractive mappings, athmandu Univ. J. Sci, Eng and Tech, 5 (2009) ,16-13 .
[25] Czerwik, S., Nonlinear set-valued contration mappings in b-metric spaces, Atti Sem Math Fis Univ Modena. 46(2), 263-276,(1998).
[26] Fi Shatanawi, Common fixed points of almost generalized (ψ, φ) s – contractive mappings in ordered b-metric spaces, Fixed Point Theory and Application(2013).
[27] Brinde .V.Iterative Approximation of Fixed Points, Springer, (2006).
[28] Ramakant Bhardwaj, Shinjani Singh "Couple soft fixed point theorems in partially ordered soft Metric spaces" "International Journal of Theoretical & Applied Sciences, 9(2)157-160(2017) UGC Approved No. 63302)
[29] Ramakant Bhardwaj, Sharad Gupta , Fixed point theorems of soft contractive mapping, (IJATER) ISSN No: 2250-3536 Vol. 6, Nov. 2016, pp 07-16 (UGC Approved No 4745)
[30] Reena M Patel, Ramakant Bhardwaj, Sunil Garge, "Some fixed point and common fixed point theorems in l- spaces for integral type mappings" International Journal of Advanced Technology & Engineering Research (IJATER) Volume 5, Issue 6, Nov. (2015), 43-54
[31] Renu Praveen Pathak, Ramakant Bhardwaj, Sunil Garge, "Generalization of Weak Contraction Mapping In Modular Space For Integral type. "International Journal of Theoretical & Applied Sciences 6(1): (2014) 61-67.
[32] Rajesh Shrivastava, Manish Sharma, Ramakant Bhardwaj, "Fixed Point Results in G-Metric Spaces Through Rational Contractive Conditions", International Journal of Theoretical & Applied Science, Val. 6, No. 2, 2014, PP.82-88.
[33] Shrivastava,R., ,Singhvi, S., Bhardwaj, R.K.,Patkar,S " Some fixed point and common fixed point theorems in Banach spaces for rational expressions. "International Journal of Theoretical & applied Sciences 3(2) (2011) 104-115.


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Implementation Of Soft Fixed Point Theorems With B-Metric Space

Kanchan Mishra, Chitra Singh, Anurag Choubey

Abstract: In this paper we instigate the different types of soft sets and different methods to solve and show some fixed point theorems on a soft metric space by using new type rational contraction conditions. Our results increase and raise the recent ones.

Keyword: b-metric space, Fixed point, Contraction mappings, Cauchy sequence, Complete Metric space, Soft set. AMS Classification: 45H10.

1. Introduction and Preliminaries:

The concept of soft sets was first given by Molodtsov [1] soft sets was new mathematical idea to solve problems. Molodtsov[1] defines soft set as a parameterized family of subsets of whole set where each element is regarded as a set of closer elements of the soft set. In the past few years, many researchers have done their researches on the fundamentals of soft set theory. Maji et al. [2] presented a OR . They researched on it and described the basic properties of there operations Pei and Miao [3] discussed the relationship between soft sets and information systems and also redefined subset and intersection of soft sets. Ali et al. [4] gave new operations like the restricted union, the restricted intersection, the restricted difference and the extended intersection of two soft sets and said emphasis mainly on their basic properties. Cagman and Enginoglu [5] introduced soft matrix theory and were successfully applying it to a decision making problem. Babitha and Sunil [6] introduced the idea of soft set relation and function and discussed some related concepts like equivalence soft set relation, partition of soft sets, ordering on soft sets. They specially worked on soft set relation and ordering. Babitha and Sunil [7] By introducing the concept of relation and got proofs with some results. Sezgin and Atagun [8], Fuli [9] and others modified the work of Maji et al. [2] and gave some new results Sezgin also put forth some new results. Sezgin and Atagun [8] gave restricted symmetric difference of soft sets and studied its properties with examples. Singh and Onyeozili [2012] got some results on distributive and absorption properties with respect to different operations on soft sets and [19, 20,21,22] added some more results of soft fixed point in b-metric space.

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Definition 2.1: Let (Y, d) is a metric space and the set (F, B) is a soft set of Y is called

null soft set and it is denoted by φ .if for all $\varepsilon \in B$, $F(\varepsilon) = \phi$. (empty set)

Definition 2.2: Let (Y, d) is a metric space and suppose that the set (F, B) is a soft set

over (Y, d) is called absolute soft set. If for all $\varepsilon \in B$, $F(\varepsilon) = Y$.

Definition 2.3: Let (Y, d) is a metric space and if the difference (F, A) of two soft sets

(F, A) and (F, A) over Y is denoted by (F, A)/ (F, A) and is defined as

$F(e) = H(e) \backslash G(e)$ for all $e \in A$.

Definition 2.4: For any two soft real numbers

(i) $\tilde{a} \leq \tilde{b}$, if $\tilde{a}(e) \leq \tilde{b}(e)$, for all $e \in A$.

(ii) $\tilde{a} \geq \tilde{b}$, if $\tilde{a}(e) \geq \tilde{b}(e)$, for all $e \in A$.

(iii) $\tilde{a} < \tilde{b}$, if $\tilde{a}(e) < \tilde{b}(e)$, for all $e \in A$.

(iv) $\tilde{a} > \tilde{b}$, if $\tilde{a}(e) > \tilde{b}(e)$, for all $e \in A$.

Definition 2.5: The complement of a soft set (F, A) is denoted by $(F, A)^c$ and is defined by

$(F, A)^c = (F^c, A)$ where $F^c : A \rightarrow P(X)$ is mapping given by $(\beta) = X - F(\beta)$,$\forall \beta \in A$.

Definition 2.6: Let R be the set of real numbers and B(R) be the collection of all nonempty

Bounded subsets of R and B taken as a set of parameters. Then a mapping

$F: H \rightarrow A(R)$ is said to be a soft real set. It is denoted by (F, H).If specifically (F, H)

is a singleton soft set, then identifying (F,H) with the corresponding soft element,

it will be said to a soft real number and denoted $\tilde{a}, \tilde{b}$ etc.

Let $\bar{0}, \bar{1}$ are any two soft real numbers where $\bar{0}(e) = 0$, $\bar{1}(e) = 1$for all $e \in H$, respectively

Definition 2.7: Let (Y, d) is a metric space and any soft set of Y is called a soft point of

Y if there is exactly one e ∈ H, such that B (e) = {y} for some y ∈ Y and

B (e′) = ϕ, ∀ e′ ∈ H \ {e}. It will be denoted by $\tilde{y}_e$.

Definition 2.8: Let (F, A) is any soft set and let any two soft point a, b of (F, A) are said

to be equal if e = e′ and B(e) = B(e′) i.e. a = b. Thus a ≠ b ⇔ a ≠ b or e ≠ e′.

Definition 2.9: A mapping $\tilde{d}$: SP ($\tilde{Y}$) × SP ($\tilde{Y}$) →R(A)*, is said to be a soft metric on the

soft set $\tilde{Y}$ if $\tilde{d}$ satisfies the following conditions:

(M1) $\tilde{d}\left(\tilde{y}_{e_1}\, \tilde{x}_{e_2}\right) \tilde{\geq} \tilde{0}$ for all $\tilde{x}_{e_1}\, \tilde{y}_{e_2} \tilde{\epsilon}\ \tilde{Y}$

(M2) $\tilde{d}\left(\tilde{y}_{e_1}\, \tilde{x}_{e_2}\right) = \tilde{0} \Leftrightarrow \tilde{x}_{e_1} = \tilde{y}_{e_2}$

(M3) $\tilde{d}\left(\tilde{x}_{e_1}\, \tilde{y}_{e_2}\right) \tilde{\geq} \tilde{d}\left(\tilde{y}_{e_2}\, \tilde{x}_{e_1}\right)$ for all $\tilde{x}_{e_1}\, \tilde{y}_{e_2} \tilde{\epsilon}\ \tilde{Y}$

(M4) $\tilde{d}\left(\tilde{x}_{e_1}\, \tilde{z}_{e_2}\right) \tilde{\geq} \tilde{d}\left(\tilde{x}_{e_1}\, \tilde{y}_{e_2}\right) + \tilde{d}\left(\tilde{y}_{e_2}\, z_{e_2}\right)$

for all $\tilde{x}_{e_1}\, \tilde{y}_{e_2}, \tilde{z}_{e_2} \in \tilde{Y}$ The soft set $\tilde{Y}$ with a soft metric d on $\tilde{Y}$ is called a soft metric space
and denoted by ($\tilde{Y}$, $\tilde{d}$ A).

Definition 2.10: A sequence $\{\tilde{y}_{\lambda_n}\}_n$ of soft point in($Y, \bar{d}$ H) is considered as a Cauchy

sequence in $\tilde{Y}$ if corresponding to every $\tilde{\epsilon} \tilde{\geq} \bar{0}$, ∃ n ∈ N such that

$d(\tilde{y}_{\lambda_i}, \tilde{y}_{\lambda_j}) \tilde{\leq} \tilde{\epsilon}$, ∀i, j ≥ n,i.e. $d(\tilde{y}_{\lambda_i}, \tilde{y}_{\lambda_j}) \to \bar{0}$, as i,j → ∞.

Definition 2.11: A soft metric space ($\tilde{Y}$, $\tilde{d}$ H) is said to be complete, if every Cauchy

Sequence in $\tilde{Y}$ converges to some point of $\tilde{Y}$.

Definition 2.12: A mapping T: Y → Y where(Y, d) is a metric space is said to be weakly

C-contractive or a weak C-contraction if for all x, y ∈ Y.

$d(T_x, T_y) \leq [d(x, T_y) + d(y, T_x)] - \xi\ d(x, T_y)$, $d(y, T_x)$ Where ξ :[0,∞)→[0,∞) is a

continuous mapping such that ξ (x, y) = 0 ⇔ x = y = 0. We introduce

the new notion of a soft weak C-contractive mapping as follow.

Definition 2.13: Let $\tilde{A}$ be a nonempty set and let $\tilde{r} \geq 1$ be a given real number. A function

$\tilde{d} : \tilde{A} \times \tilde{A} \to \tilde{R}^+$ is called a b-metric provided that, for $a, \tilde{b}, \tilde{c} \in \tilde{A}$

(a) $\tilde{d}(\tilde{a}, \tilde{b}) = 0 \Leftrightarrow \tilde{a} = \tilde{b}$

(b) $\tilde{d}(\tilde{a}, \tilde{b}) = \tilde{d}(\tilde{b}, \tilde{a})$

(c) $\tilde{d}(\tilde{a}, \tilde{c}) \leq \tilde{s}[\tilde{d}(\tilde{a}, \tilde{b}) + \tilde{d}(\tilde{b}, \tilde{c})]$.

3. Main Result:

Let ($\tilde{Y}$, $\tilde{d}$) be a soft complete b-metric space with the constant $\tilde{s} \geq 1$ and define the sequence

$\{y_n\}_{n=1}^{\infty} \subset K$ by the recursion(2.13). Let $(f, \xi): \tilde{Y} \to \tilde{Y}$ be a mapping for which there exists

$\mu \in [0.\frac{1}{2})$ such that

$\tilde{d}[(f, \xi)\tilde{x}, (f, \xi)\tilde{y}] \leq \mu[\tilde{d}(\tilde{x}, (f, \xi)\tilde{x}) + \tilde{d}(\tilde{y}, (f, \xi)\tilde{y})]$ (A)

For all $\tilde{x}, \tilde{y} \in \tilde{Y}$.

Theorem 3.1: Suppose ($\tilde{Y}$, $\tilde{d}$) be a soft complete b-metric space with the constant $s \geq 1$ and explain the sequence $\{y_n\}_{n=1}^{\infty} \subset K$ by (2.13). Let $(f, \xi): \tilde{Y} \to \tilde{Y}$ be a contraction with the reduction $K \in [0.1)$, and $Ks < 1$. Then there exists $\tilde{y}_\omega \in \tilde{Y}$ such that $\tilde{y}_\omega^n \to \tilde{y}_\omega$ and $\tilde{y}_\omega$ is unique soft fixed point of (f, ξ).

Proof: Let $\tilde{y}_\omega^0 \in \tilde{Y}$ be any arbitrary chosen soft point in $\tilde{Y}$ and $\{y_n\}_{n=1}^{\infty}$ be a sequence

Let $\tilde{y}_{\omega n}^n = (f, \xi)\tilde{y}_{\omega n-1}^{n-1} = (f, \xi)^n \tilde{y}_\omega^0$, n=1, 2, _ _ _ _

Since (f, ξ) is a contraction mapping with the constant $K \in [0, 1)$

Then we get

$\tilde{d}[(f, \xi)^2 \tilde{y}_\omega^0, (f, \xi)^2 \tilde{y}_{\omega 1}^1] \leq K\, \tilde{d}[(f, \xi)\tilde{y}_\omega^0, (f\xi,)\, \tilde{y}_{\omega 1}^1]$

$\leq K^2\, \tilde{d}[\tilde{y}_\omega^0, y_{\omega 1}^1]$

We easily get here

$\tilde{d}[(f, \xi)^2 \tilde{y}_\omega^0, (f, \xi)^2\ \tilde{y}_{\omega 1}^1] \leq K^2\, \tilde{d}[\tilde{y}_\omega^0, \tilde{y}_{\omega 1}^1]$

Now, we proof that $\{y_n\}_{n=1}^{\infty}$ is a Cauchy sequence in $\tilde{Y}$.

Suppose that m, n > 0 with m > n,

$\tilde{d}(\tilde{y}_{\omega n}^n, \tilde{y}_{\omega m}^m) < p\, \tilde{d}(\tilde{y}_{\omega n}^n, \tilde{y}_{\omega n+1}^{n+1}) + p^2\, \tilde{d}(\tilde{y}_{\omega n+1}^{n+1}, \tilde{y}_{\omega n+1}^{n+1}) + p^3\, \tilde{d}(\tilde{y}_{\omega n+2}^{n+2}, \tilde{y}_{\omega n+3}^{n+3})$-------

$\leq pK^n\, \tilde{d}(\tilde{y}_\omega^0, \tilde{y}_{\omega 1}^1) + p^2 K^{n+1}\, \tilde{d}(\tilde{y}_\omega^0, \tilde{y}_{\omega 1}^1) + p^3 K^{n+2}\, \tilde{d}(\tilde{y}_\omega^0, \tilde{y}_{\omega 1}^1)$++----------

$= \tilde{d}(\tilde{y}_\omega^0, \tilde{y}_\omega^1)\, pK^n\, [1 + pK + (\, pK^2 + pK^3 + ++)\,]$ (3.1.1)

Now we take $n, m \to \infty$ in (3.1.1), we get here at $\lim_{n\to\infty} \tilde{d}(\tilde{y}^{m}_{\omega m}, \tilde{y}^{n}_{\omega n}) = 0$

Therefor $\{y_n\}_{n=1}^{\infty}$ is a Cauchy sequence in $\tilde{Y}$.

In view of completeness of $\tilde{Y}$. We examine that $\{y_n\}_{n=1}^{\infty}$ convergent to $y_\omega \in \tilde{Y}$.

Now we proof that $\tilde{y}_\omega$ is the unique fixed point of (f, ξ).

Actually $(f,\xi)\ \tilde{y}_\omega = \lim_{n\to\infty}(f,\xi)\tilde{y}^{n}_{\omega n} = \lim_{n\to\infty}(f,\xi)\ \tilde{y}^{n+1}_{\omega n+1}$

$= \tilde{y}_\omega$

Therefore $\tilde{y}_\omega$ is a soft fixed point of (f, ξ). Finally we have to

Proof that the soft fixed point is unique.

Suppose that $\tilde{y}'$ is another soft fixed point of (f, ξ). Then $(f,\xi)\tilde{y}' = \tilde{y}'$

$$\tilde{d}(\tilde{y}_\omega, \tilde{y}') = \tilde{d}[(f,\xi)\tilde{y}_\omega, (f,\xi)y'] \le K\,\tilde{d}(\tilde{y}_\omega, \tilde{y}') \qquad (3.1.2)$$

By equation (4) $K \ge 1$; but this is a contradiction to $K \in [0,1)$.

So the soft fixed point is unique.

Theorem 3.2: Let $(\tilde{Y}, \tilde{d})$ be a soft complete b-metric space with the constant $s \ge 1$ and

define the sequence $\{y_n\}_{n=1}^{\infty} \subset K$ by the (2.13). Let $(f,\xi): \tilde{Y} \to \tilde{Y}$ be a

mapping for which there exists $\mu \in [0, \frac{1}{2})$ such that

$$\tilde{d}[(f,\xi)\tilde{x}, (f,\xi)\tilde{y}] \le \mu[\tilde{d}(\tilde{x}, (f,\xi)\tilde{x}) + \tilde{d}(\tilde{y}, (f,\xi)\tilde{y})] + \delta[\tilde{d}(\tilde{x}, (f,\xi)\tilde{y}) + \tilde{d}(\tilde{y}, (f,\xi)\tilde{x})] + \eta\,\tilde{d}(\tilde{x}, \tilde{y}) + \beta\left[\frac{\tilde{d}\{(\tilde{x}, (f,\xi)\tilde{y}) + \tilde{d}(\tilde{y}, (f,\xi)\tilde{x})\}}{1+\tilde{d}\{(\tilde{x}, (f,\xi)\tilde{x})\}\tilde{d}\{(\tilde{y}, (f,\xi)\tilde{y})\}\tilde{d}\{(\tilde{y}, (f,\xi)\tilde{x})\}\tilde{d}\{(\tilde{x}, (f,\xi)\tilde{y})\}}\right] \qquad (3.2.1)$$

For all $\tilde{x}, \tilde{y} \in \tilde{Y}$.

Then there exists $\tilde{y}'_\omega \in \tilde{Y}$ such that $\tilde{y}_{\omega n} \to \tilde{y}'_\omega$ and $\tilde{y}'_\omega$ is unique fixed point of (f, ξ).

Proof: Suppose $\tilde{y}^{0}_{\omega} \in \tilde{Y}$ and $\{y_n\}_{n=1}^{\infty}$ be a sequence in $\tilde{Y}$.

$\tilde{y}^{n}_{\omega n} = (f,\xi)\tilde{y}^{n-1}_{\omega n-1} = (f,\xi)^n y^{0}_{\omega}$, n=1, 2, 3 _ _ _ _

By using equation (A) and (2.13). We obtain that

$$\tilde{d}(\tilde{y}^{n}_{\omega n}, \tilde{y}^{n+1}_{\omega n+1}) = \tilde{d}\{(f,\xi)\tilde{y}^{n-1}_{\omega n-1}, (f,\xi)\tilde{y}^{n}_{\omega n}\}$$

$$\tilde{d}(\tilde{y}^{n}_{\omega n}, \tilde{y}^{n+1}_{\omega n+1}) \le \mu[\tilde{d}(\tilde{y}^{n-1}_{\omega n-1}, (f,\xi)\tilde{y}^{n-1}_{\omega n-1}) + \tilde{d}(\tilde{y}^{n}_{\omega n}, (f,\xi)\tilde{y}^{n}_{\omega n}] + \delta\,\tilde{d}(\tilde{y}^{n-1}_{\omega n-1}, (f,\xi)\tilde{y}^{n}_{\omega n}) + \eta\tilde{d}(\tilde{y}^{n-1}_{\omega n-1}, \tilde{y}^{n}_{\omega n}) + \beta\left[\frac{\tilde{d}\{\tilde{y}^{n-1}_{\omega n-1}, (f,\xi)\tilde{y}^{n}_{\omega n}\} + \tilde{d}\{(f,\xi)\tilde{y}^{n-1}_{\omega n-1}, \tilde{y}^{n}_{\omega n}\}}{1+\tilde{d}\{(\tilde{y}^{n-1}_{\omega n-1}, (f,\xi)\tilde{y}^{n-1}_{\omega n-1})\}\tilde{d}\{(\tilde{y}^{n}_{\omega n}, (f,\xi)\tilde{y}^{n}_{\omega n})\}\tilde{d}\{((f,\xi)\tilde{y}^{n-1}_{\omega n-1}, \tilde{y}^{n}_{\omega n})\}\tilde{d}\{(\tilde{y}^{n-1}_{\omega n-1}, (f,\xi)\tilde{y}^{n}_{\omega n})\}}\right] \qquad (3.2.3)$$

$$\tilde{d}(\tilde{y}^{n}_{\omega n}, \tilde{y}^{n+1}_{\omega n+1}) \le \mu[\tilde{d}(\tilde{y}^{n-1}_{\omega n-1}, \tilde{y}^{n}_{\omega n}) + \tilde{d}(\tilde{y}^{n}_{\omega n}, \tilde{y}^{n+1}_{\omega n+1})] + \delta[\tilde{d}(\tilde{y}^{n-1}_{\omega n-1}, \tilde{y}^{n}_{\omega n}) + \tilde{d}(\tilde{y}^{n}_{\omega n}, \tilde{y}^{n+1}_{\omega n+1})] + \eta\,\tilde{d}(\tilde{y}^{n-1}_{\omega n-1}, \tilde{y}^{n}_{\omega n}) + \beta[\tilde{d}(\tilde{y}^{n-1}_{\omega n-1}, \tilde{y}^{n}_{\omega n}) + \tilde{d}(\tilde{y}^{n}_{\omega n}, \tilde{y}^{n+1}_{\omega n+1})]$$

$$\tilde{d}(\tilde{y}^{n}_{\omega n}, \tilde{y}^{n+1}_{\omega n+1}) \le (\mu+\delta+\eta+\beta)\,\tilde{d}(\tilde{y}^{n-1}_{\omega n-1}, \tilde{y}^{n}_{\omega n}) + (\mu+\delta+\beta)\tilde{d}(\tilde{y}^{n}_{\omega n}, \tilde{y}^{n+1}_{\omega n+1})$$

$$[1-(\mu+\delta+\beta)]\tilde{d}(\tilde{y}^{n}_{\omega n}, \tilde{y}^{n+1}_{\omega n+1}) \le (\mu+\delta+\eta+\beta)\,\tilde{d}(\tilde{y}^{n-1}_{\omega n-1}, \tilde{y}^{n}_{\omega n})$$

$$\tilde{d}(\tilde{y}^{n}_{\omega n}, \tilde{y}^{n+1}_{\omega n+1}) \le \left[\frac{\mu+\delta+\eta+\beta}{\mu+\delta+\beta}\right]\tilde{d}(\tilde{y}^{n-1}_{\omega n-1}, \tilde{y}^{n}_{\omega n})$$

Similarly $\tilde{d}(\tilde{y}^{n}_{\omega n}, \tilde{y}^{n+1}_{\omega n+1}) \le \left[\frac{\mu+\delta+\eta+\beta}{\mu+\delta+\beta}\right]\tilde{d}(\tilde{y}^{0}_{\omega}, \tilde{y}^{1}_{\omega 1})$ (3.2.2)

Note that $\mu \in [0, \frac{1}{2})$ Then $\left[\frac{\mu+\delta+\eta+\beta}{\mu+\delta+\beta}\right]^n \in [0, \frac{1}{2})$

Thus (f,ξ) is a contraction mapping. We know that $\{y_n\}_{n=1}^{\infty}$ is a Cauchy sequence and convergent sequence, It is convergent to $\tilde{y}_\omega \in \tilde{Y}$.

Then we have

$$\tilde{d}(\tilde{y}^{*}_{\omega}, (f,\xi)\tilde{y}^{*}_{\omega}) \le s[\tilde{d}(\tilde{y}^{*}_{\omega}, \tilde{y}^{n+1}_{\omega n+1}) + \tilde{d}(\tilde{y}^{n+1}_{\omega n+1}, \tilde{y}^{n}_{\omega n}) + \tilde{d}(\tilde{y}^{n}_{\omega n}, (f,\xi)\tilde{y}^{*}_{\omega n})]$$

$$\le s[\tilde{d}(\tilde{y}^{*}_{\omega}, \tilde{y}^{n+1}_{\omega n+1}) + \tilde{d}(\tilde{y}^{n+1}_{\omega n+1}, \tilde{y}^{n}_{\omega n}) + \tilde{d}((f,\xi)\tilde{y}^{n-1}_{\omega n-1}, (f,\xi)\tilde{y}^{*}_{\omega n})]$$

$$\tilde{d}(\tilde{y}^{n}_{\omega n}, \tilde{y}^{n+1}_{\omega n+1}) \le \mu[\tilde{d}(\tilde{y}^{n-1}_{\omega n-1}, \tilde{y}^{n}_{\omega n}) + \tilde{d}(\tilde{y}^{n}_{\omega n}, \tilde{y}^{n+1}_{\omega n+1}] + \delta\,\tilde{d}(\tilde{y}^{n-1}_{\omega n-1}, \tilde{y}^{n+1}_{\omega n+1}) + \eta\tilde{d}(\tilde{y}^{n-1}_{\omega n-1}, \tilde{y}^{n}_{\omega n}) + \beta\left[\frac{\tilde{d}\{\tilde{y}^{n-1}_{\omega n-1}, \tilde{y}^{n+1}_{\omega n+1}\} + \tilde{d}\{\tilde{y}^{n}_{\omega n}, \tilde{y}^{n}_{\omega n}\}}{1+\tilde{d}\{(\tilde{y}^{n-1}_{\omega n-1}, \tilde{y}^{n}_{\omega n})\}\tilde{d}\{(\tilde{y}^{n}_{\omega n}, \tilde{y}^{n+1}_{\omega n+1})\}\tilde{d}\{(\tilde{y}^{n}_{\omega n}, \tilde{y}^{n}_{\omega n})\}\tilde{d}\{\tilde{y}^{n-1}_{\omega n-1}, \tilde{y}^{n+1}_{\omega n+1}\}}\right]$$

$\leq$

$$s\left[\begin{array}{c}\tilde{d}(\tilde{y}^{*}_{\omega},\tilde{y}^{n+1}_{\omega n+1})\,\tilde{d}(\tilde{y}^{n+1}_{\omega n+1},\tilde{y}^{n}_{\omega n})\\ +\mu\{\tilde{d}(\tilde{y}^{n-1}_{\omega n-1},(f,\xi)\tilde{y}^{n-1}_{\omega n-1})+\tilde{d}(\tilde{y}^{*}_{\omega n},(f,\xi)\tilde{y}^{*}_{\omega})\}\\ +\delta\{\tilde{d}(\tilde{y}^{n-1}_{\omega n-1},(f,\xi)\tilde{y}^{*}_{\omega})+\tilde{d}(\tilde{y}^{*}_{\omega},(f,\xi)\tilde{y}^{n-1}_{\omega n-1})\}+\eta\,\tilde{d}(\tilde{y}^{n-1}_{\omega n-1},\tilde{y}^{*}_{\omega})\\ +\beta\left[\frac{\{\tilde{d}(\tilde{y}^{n-1}_{\omega n-1},(f,\xi)\tilde{y}^{*}_{\omega})+\tilde{d}(\tilde{y}^{*}_{\omega},(f,\xi)\tilde{y}^{n-1}_{\omega n-1})\}}{1+\tilde{d}(\tilde{y}^{n-1}_{\omega n-1},(f,\xi)\tilde{y}^{n-1}_{\omega n-1})\,\tilde{d}(\tilde{y}^{*}_{\omega},(f,\xi)\tilde{y}^{*}_{\omega})\,\tilde{d}(\tilde{y}^{n-1}_{\omega n-1},(f,\xi)\tilde{y}^{*}_{\omega})\,\tilde{d}(\tilde{y}^{*}_{\omega},(f,\xi)\tilde{y}^{n-1}_{\omega n-1})}\right]\end{array}\right]$$

$\tilde{d}(\tilde{y}^{*}_{\omega},(f,\xi)\tilde{y}^{*}_{\omega})$

$$\leq s\left[\begin{array}{c}\tilde{d}(\tilde{y}^{*}_{\omega},\tilde{y}^{n+1}_{\omega n+1})\,\tilde{d}(\tilde{y}^{n+1}_{\omega n+1},\tilde{y}^{n}_{\omega n})\\ +\mu\{\tilde{d}(\tilde{y}^{n-1}_{\omega n-1},\tilde{y}^{n}_{\omega n})+\tilde{d}(\tilde{y}^{*}_{\omega n},(f,\xi)\tilde{y}^{*}_{\omega})\}\\ +\delta\{\tilde{d}(\tilde{y}^{n-1}_{\omega n-1},(f,\xi)\tilde{y}^{*}_{\omega})+\tilde{d}(\tilde{y}^{*}_{\omega},\tilde{y}^{n}_{\omega n})\}+\eta\,\tilde{d}(\tilde{y}^{n-1}_{\omega n-1},\tilde{y}^{*}_{\omega})\\ +\beta\left[\frac{\{\tilde{d}(\tilde{y}^{n-1}_{\omega n-1},(f,\xi)\tilde{y}^{*}_{\omega})+\tilde{d}(\tilde{y}^{*}_{\omega},(f,\xi)\tilde{y}^{n}_{\omega n})\}}{1+\tilde{d}(\tilde{y}^{n-1}_{\omega n-1},\tilde{y}^{n}_{\omega n})\,\tilde{d}(\tilde{y}^{*}_{\omega},(f,\xi)\tilde{y}^{*}_{\omega})\,\tilde{d}(\tilde{y}^{n-1}_{\omega n-1},(f,\xi)\tilde{y}^{*}_{\omega})\,\tilde{d}(\tilde{y}^{*}_{\omega},\tilde{y}^{n}_{\omega n})}\right]\end{array}\right]$$

$$(1-s\mu)\,\tilde{d}(\tilde{y}^{*}_{\omega},(f,\xi)\tilde{y}^{*}_{\omega})\leq$$

$$s\left[\begin{array}{c}\tilde{d}(\tilde{y}^{*}_{\omega},\tilde{y}^{n+1}_{\omega n+1})+\tilde{d}(\tilde{y}^{n+1}_{\omega n+1},\tilde{y}^{n}_{\omega n})+\mu\,\tilde{d}(\tilde{y}^{n-1}_{\omega n-1},\tilde{y}^{n}_{\omega n})\\ +\delta\{\tilde{d}(\tilde{y}^{n-1}_{\omega n-1},(f,\xi)\tilde{y}^{*}_{\omega})+\tilde{d}(\tilde{y}^{*}_{\omega},\tilde{y}^{*}_{\omega n})\}+\eta\,\tilde{d}(\tilde{y}^{n-1}_{\omega n-1},\tilde{y}^{*}_{\omega})\\ +\beta\left[\frac{\{\tilde{d}(\tilde{y}^{n-1}_{\omega n-1},(f,\xi)\tilde{y}^{*}_{\omega})+\tilde{d}(\tilde{y}^{*}_{\omega},(f,\varphi)\tilde{y}^{n}_{\omega n})\}}{1+\tilde{d}(\tilde{y}^{n-1}_{\omega n-1},\tilde{y}^{n}_{\omega n})\,\tilde{d}(\tilde{y}^{*}_{\omega},(f,\xi)\tilde{y}^{*}_{\omega})\,\tilde{d}(\tilde{y}^{n-1}_{\omega n-1},(f,\xi)\tilde{y}^{*}_{\omega})\,\tilde{d}(\tilde{y}^{*}_{\omega},\tilde{y}^{n}_{\omega n})}\right]\end{array}\right]$$

$$\tilde{d}(\tilde{y}^{*}_{\omega},(f,\xi)\tilde{y}^{*}_{\omega})\leq$$

$$\left[\frac{s}{1-s\mu}\right]\left[\begin{array}{c}\tilde{d}(\tilde{y}^{*}_{\omega},\tilde{y}^{n+1}_{\omega n+1})+\tilde{d}(\tilde{y}^{n+1}_{\omega n+1},\tilde{y}^{n}_{\omega n})+\mu\,\tilde{d}(\tilde{y}^{n-1}_{\omega n-1},\tilde{y}^{n}_{\omega n})\\ +\delta\{\tilde{d}(\tilde{y}^{n-1}_{\omega n-1},(f,\varphi)\tilde{y}^{*}_{\omega})+\tilde{d}(\tilde{y}^{*}_{\omega},\tilde{y}^{*}_{\omega n})\}+\eta\,\tilde{d}(\tilde{y}^{n-1}_{\omega n-1},\tilde{y}^{*}_{\omega})\\ +\beta\left[\frac{\{\tilde{d}(\tilde{y}^{n-1}_{\omega n-1},(f,\xi)\tilde{y}^{*}_{\omega})+\tilde{d}(\tilde{y}^{*}_{\omega},(f,\xi)\tilde{y}^{n}_{\omega n})\}}{1+\tilde{d}(\tilde{y}^{n-1}_{\omega n-1},\tilde{y}^{n}_{\omega n})\,\tilde{d}(\tilde{y}^{*}_{\omega},(f,\xi)\tilde{y}^{*}_{\omega})\,\tilde{d}(\tilde{y}^{n-1}_{\omega n-1},(f,\xi)\tilde{y}^{*}_{\omega})\,\tilde{d}(\tilde{y}^{*}_{\omega},\tilde{y}^{n}_{\omega n})}\right]\end{array}\right]$$

Let lim $n\to\infty$, $\lim_{n\to\infty}\tilde{d}\,(\tilde{y}^{*}_{\omega},(f,\xi)\tilde{y}^{*}_{\omega})=0$

$\therefore\ \tilde{y}^{*}_{\omega}=(f,\xi)\tilde{y}^{*}_{\omega}$

$\tilde{y}^{*}_{\omega}$ is the fixed point of (f,ξ).

Theorem 3.3: Let $(\tilde{Y},\tilde{d})$ be a soft complete b-metric space and define the sequence

$\{y_n\}_{n=1}^{\infty}\subset K$ by (2.13). Let $(f,\xi):\tilde{Y}\to\tilde{Y}$ be a mapping for which there exists $\mu\in[0,\frac{1}{2})$ such that $\tilde{d}[(f,\xi)\tilde{x},(f,\xi)\tilde{y}]\leq\lambda[\tilde{d}(\tilde{x},(f,\xi)\tilde{y})+\tilde{d}(\tilde{y},(f,\xi)\tilde{x})]$

For all $\tilde{x},\tilde{y}\in\tilde{Y}$.

Proof: Suppose that $\tilde{x}^{0}_{v}\in\tilde{Y}$ and $\{x_n\}_{n=1}^{\infty}$ be a sequence in $\tilde{Y}$, defined as

$\tilde{x}^{n}_{vn}=(f,\xi)\tilde{x}^{n-1}_{vn-1}=(f,\xi)^{n}\tilde{x}^{0}_{v}$, n = 1, 2, 3 _ _ _ _

$\tilde{d}(\tilde{x}^{n+1}_{vn+1},\tilde{x}^{n+2}_{vn+2})=\tilde{d}\{(f,\xi)\tilde{x}^{n}_{vn},(f,\xi)\tilde{x}^{n+1}_{vn+1}\}$

$\tilde{d}(\tilde{x}^{n+1}_{vn+1},\tilde{x}^{n+2}_{vn+2})\leq s[\tilde{d}(\tilde{x}^{n}_{vn},(f,\xi)\tilde{x}^{n+1}_{vn+1})+\tilde{d}(\tilde{x}^{n+1}_{vn+1},(f,\xi)\tilde{x}^{n}_{vn}]$

$\leq s[\tilde{d}(\tilde{x}^{n}_{vn},\tilde{x}^{n+2}_{vn+2})+\tilde{d}(\tilde{x}^{n+1}_{vn+1},\tilde{x}^{n+1}_{vn+1}]$

$\tilde{d}(\tilde{x}^{n+1}_{vn+1},\tilde{x}^{n+2}_{vn+2})\leq\lambda[\tilde{d}(\tilde{x}^{n}_{vn},\tilde{x}^{n+2}_{vn+2})]$

$\tilde{d}(\tilde{x}^{n+1}_{vn+1},\tilde{x}^{n+2}_{vn+2})\leq\lambda[\tilde{d}(\tilde{x}^{n}_{vn},\tilde{x}^{n+1}_{vn+1})+\tilde{d}(\tilde{x}^{n+1}_{vn+1},\tilde{x}^{n+2}_{vn+2})]$ (by 2.13)

$(1-s\lambda)\,\tilde{d}(\tilde{x}^{n+1}_{vn+1},\tilde{x}^{n+2}_{vn+2})\leq s\lambda[\tilde{d}(\tilde{x}^{n}_{vn},\tilde{x}^{n+1}_{vn+1})]$

$\tilde{d}(\tilde{x}^{n+1}_{vn+1},\tilde{x}^{n+2}_{vn+2})\leq\left[\frac{s\lambda}{1-s\lambda}\right]\tilde{d}(\tilde{x}^{n}_{vn},\tilde{x}^{n+1}_{vn+1})$

Note that $\left[\frac{s\lambda}{1-s\lambda}\right]\in[0,\frac{1}{2})$ (3.3.1)

Thus (f,ξ) is contraction mapping, by Theorem 3.1, We See that

$\{y_n\}_{n=1}^{\infty}$ is a Cauchy sequence and It is a convergent sequence.

Now we show that $\tilde{x}^{*}_{v}$ is a soft fixed point of (f,ξ).

$\tilde{d}(\tilde{x}^{*}_{v},(f,\xi)\tilde{x}^{*}_{v})\leq s\,[\tilde{d}(\tilde{x}^{*}_{v},\tilde{x}^{n}_{vn})+\tilde{d}(\tilde{x}^{n}_{vn},(f,\xi)\tilde{x}^{*}_{v})]$

$\leq s[\tilde{d}(\tilde{x}^{*}_{v},\tilde{x}^{n}_{vn})+\tilde{d}\{(f,\xi)\tilde{x}^{n-1}_{vn-1},(f,\xi)\tilde{x}^{*}_{v}\}]$

$$\leq s\left[\begin{array}{c}\tilde{d}(\tilde{x}^{*}_{v},\tilde{x}^{n}_{vn})+\mu\{\tilde{d}(\tilde{x}^{n-1}_{vn-1},(f,\xi)\tilde{x}^{n-1}_{vn-1})+\tilde{d}(\tilde{x}^{*}_{v},(f,\xi)\tilde{x}^{*}_{v})\}\\ +\delta\,\tilde{d}(\tilde{x}^{n-1}_{vn-1},(f,\xi)\tilde{x}^{*}_{v})+\eta\,\tilde{d}(\tilde{x}^{n-1}_{vn-1},\tilde{x}^{*}_{v})\\ +\beta\left[\frac{\{\tilde{d}(\tilde{x}^{n-1}_{vn-1},(f,\xi)\tilde{x}^{*}_{v})+\tilde{d}(\tilde{x}^{*}_{v},(f,\xi)\tilde{x}^{n-1}_{vn-1})\}}{1+\tilde{d}(\tilde{x}^{n-1}_{vn-1},(f,\xi)\tilde{x}^{n-1}_{vn-1})\tilde{d}(\tilde{x}^{*}_{v},(f,\xi)\tilde{x}^{*}_{v})+\tilde{d}(\tilde{x}^{n-1}_{vn-1},(f,\xi)\tilde{x}^{*}_{v})\tilde{d}(\tilde{x}^{*}_{v},\tilde{x}^{n-1}_{vn-1})}\right]\end{array}\right]$$

$$\leq s\left[\begin{array}{c}\tilde{d}(\tilde{x}^{*}_{v},\tilde{x}^{n}_{vn})+\mu\{\tilde{d}(\tilde{x}^{n-1}_{vn-1},\tilde{x}^{n}_{vn})+\tilde{d}(\tilde{x}^{*}_{v},(f,\xi)\tilde{x}^{*}_{v})\}\\ +\delta\{\tilde{d}(\tilde{x}^{n-1}_{vn-1},(f,\xi)\tilde{x}^{*}_{v})+\tilde{d}(\tilde{x}^{*}_{v},\tilde{x}^{n}_{vn})\}+\eta\,\tilde{d}(\tilde{x}^{n-1}_{vn-1},\tilde{x}^{*}_{v})\\ +\beta\left[\frac{\{\tilde{d}(\tilde{x}^{n-1}_{vn-1},(f,\xi)\tilde{x}^{*}_{v})+\tilde{d}(\tilde{x}^{*}_{v},\tilde{x}^{n}_{vn})\}}{1+\tilde{d}(\tilde{x}^{n-1}_{vn-1},\tilde{x}^{n}_{vn})\,\tilde{d}(\tilde{x}^{*}_{v},(f,\xi)\tilde{x}^{*}_{v})+\tilde{d}(\tilde{x}^{n-1}_{vn-1},(f,\xi)\tilde{x}^{*}_{v})\,\tilde{d}(\tilde{x}^{*}_{v},\tilde{x}^{n}_{vn})}\right]\end{array}\right]$$

Lim $n\to\infty$, $\lim_{n\to\infty}\tilde{d}\,(\tilde{x}^{*}_{v},(f,\xi)\tilde{x}^{*}_{v})\leq s\,\mu\,\tilde{d}\,(\tilde{x}^{*}_{v},(f,\xi)\tilde{x}^{*}_{v})$ (3.3.3)

Equation (3.3.3) is false unless $\tilde{d}(\tilde{x}^{*}_{v},(f,\xi)\tilde{x}^{*}_{v})=0$

Thus we get here at $\tilde{x}^{*}_{v}=(f,\xi)\tilde{x}^{*}_{v}$

Now we proof that $\tilde{x}^{*}_{v}$ is the unique soft fixed point of (f,ξ). We suppose that $\tilde{x}^{*}_{v}$

Is another soft fixed point of (f,ξ).

Then we have $\tilde{x_1}=(f,\xi)\tilde{x_1}$, $\tilde{y_1}=(f,\xi)\tilde{y_1}$

$\tilde{d}(\tilde{x_1},\tilde{y_1})=$

$\mu\left[\tilde{d}(\tilde{x_1},(f,\xi)\tilde{x_1})+\tilde{d}(\tilde{y_1},(f,\xi)\tilde{y_1})\right]+\delta\left[\begin{array}{c}\tilde{d}(\tilde{x_1},(f,\xi)\tilde{y_1})+\\ \tilde{d}(\tilde{y_1},(f,\xi)\tilde{x_1})\end{array}\right]+\eta\,\tilde{d}(\tilde{x_1},\tilde{y_1})$

$$+\beta\left[\frac{\{\tilde{d}(\tilde{x_1},(f,\xi)\tilde{y_1})+\tilde{d}(\tilde{y_1},(f,\xi)\tilde{x_1})\}}{1+\tilde{d}(\tilde{x_1},(f,\xi)\tilde{x_1})\tilde{d}(\tilde{y_1},(f,\xi)\tilde{y_1})+\tilde{d}(\tilde{x_1},(f,\xi)\tilde{y_1})\,\tilde{d}(\tilde{y_1},(f,\xi)\tilde{x_1})}\right]$$

$\tilde{d}(\tilde{x_1},\tilde{y_1})=\mu\left[\tilde{d}(\tilde{x_1},\tilde{x_1})+\tilde{d}(\tilde{y_1},\tilde{y_1})\right]+\delta\left[\tilde{d}(\tilde{x_1},\tilde{y_1})+\tilde{d}(\tilde{y_1},\tilde{x_1})\right]+\eta\,\tilde{d}(\tilde{x_1},\tilde{y_1})$

$$+\beta\left[\frac{\{\tilde{d}(\tilde{x_1},\tilde{y_1})+\tilde{d}(\tilde{y_1},\tilde{x_1})\}}{1+\tilde{d}(\tilde{x_1},\tilde{x_1})\,\tilde{d}(\tilde{y_1},\tilde{y_1})+\tilde{d}(\tilde{x_1},\tilde{y_1})\,\tilde{d}(\tilde{y_1},\tilde{x_1})}\right]$$

$\tilde{d}(\tilde{x_1},\tilde{y_1})=\mu\left[\tilde{d}(\tilde{x_1},\tilde{x_1})+\tilde{d}(\tilde{y_1},\tilde{y_1})\right]+\delta\left[\tilde{d}(\tilde{x_1},\tilde{y_1})+\tilde{d}(\tilde{y_1},\tilde{x_1})\right]+\eta\,\tilde{d}(\tilde{x_1},\tilde{y_1})$

$+\ \beta\left[\tilde{d}(\tilde{x_1},\tilde{y_1})+\tilde{d}(\tilde{x_1},\tilde{y_1})\right]$

$\tilde{d}(\tilde{x_1},\tilde{y_1})\leq(2\delta+\eta+2\beta)\,\tilde{d}(\tilde{x_1},\tilde{y_1})$

Which is a contradiction. This implies that $x = \tilde{x}_v^*$.

This complete the proof.

CONCLUSION:

Hence in the above paper we have proved some unique fixed point theorems in soft b-metric space and we have discussed the fundamentals of soft b-metric space and also put forth different methods to solve soft b-metric space theorems.

REFERENCES

[1] Molodtsov, D. Soft –theory-first result, Comput .Math. Appl. 37, 19-31 (1999).

[2] Moji, P.K. Roy, A.R, Bisbas, R .An application of soft sets in a decision making problem, Comput. Math. Appl. 44, 1077 – 1083 (2002).

[3] Moji, P.K .Roy, A .R, Bisbas , R. Soft set theory, Computer. Math. Appl. 45, 555 – 562 (2003)

[4] Ali, M.I., Feng, F., Liu, X., Min, W.K., Shabir, M.: On some new operations in set theory. Computer. Math. Appl. 57, 1547–1553(2009).

[5] C¸a˘gman, N., Enginoglu, S.: Soft set theory uni-int decision making. Eur. J. Oper. Res. 207, 845–855 (2010).

[6] Babitha, K.V., Sunil, J. J.: Soft set relations and functions. Comput. Math., Appl. 60, 1840–1849 (2010).

[7] Sezgin, A., Atag¨un, A.O.: On operations of soft sets. Comput. Math. Appl. 61, 1457–1467 (2011).

[8] Feng, F, Liu, X, Leoreanu – Fotea , V, Jun ,YB: Soft sets and soft rough sets.Inf.Sci.181, 1125-1137 (2011).

[9] Chen.D."the parameterization reduction of soft sets and its applications" Comput. Math. Appl. 49, 757 – 763 (2010).

[10] Das. S. Samanta, S.K. "on soft metric space," J. Fuzzy Math. 21, 707-734(2013).

[11] Das. S. Samanta, S.K. Soft metric, Ann. Fuzzy Math. In form 6 , 77 – 94(2013).

[12] Das. S. Samanta, S.K. Soft real set, soft real Number and there properties. J. Fuzzy Math. 20, 551 – 576 (2012).

[13] Gunduz, C. (Aras), Sonmez, A. Cakalli, H. On soft mapping, arxiv , 1305. 4545 V1 (Math, GM) 16 may (2013).

[14] Hussain, S. Ahmad, B. Some Properties of soft topological space, Comput. Math. Apple. 62, 4058-4067(2016).

[15] Majumdar, P. and Samanta, S.K. On soft mapping Comput. Math. Appl. 60, 2666- 2672(2010).

[16] Rhoades, B.E. A comparison of various definition of contractive mappings, Trans. Amer. Math. Soc. 266, 257 – 290(1977).

[17] Rhoades, B.E. Some theorems on weakly contractive maps. Nonlinear Analysis, 47, 2683 – 2693(2001).

[18] Shabir, M. and Naz, M. On soft topological space, Comput. Math. Appl. 61, 1786- 1799 (2011).

[19] Balaji R. Wadkar, Ramakant Bhardwaj, Basant Singh, "Coupled fixed point theorems with monotone property in soft metric and soft b-metric space" International journal of Mathematical Analysis, Vol. 11, No. 1, pp 363-375, (2017).

[20] B. R. Wadkar, R. Bhardwaj, V. N. Mishra, B. Singh, "Coupled fixed point theorems in soft metric and b- soft metric space" Scientific publication of the state university of Novi Pozar, Ser. A: Appl. Math. Inform. And Mech. Vol. 9, No. 1, pp 59-73, (2017).

[21] Balaji Raghunath Wadkar, Ramakant Bhardwaj, Vishnu Narayan Mishra, Basant Singh, "Fixed point results related to soft sets" Australian Journal of Basic and Applied Sciences, Vol. 10, No. 16, Pages: 128-137, (2016)

[22] Balaji R. Wadkar, Ramakant Bhardwaj, Vishnu Narayan Mishra, Basant Singh "Fixed point theorems in dislocated Soft metric space" Open Journal of D.

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Implementation of Coupled Fixed Point Theorems with β −Monotone Property in Soft b-Metric Space

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Abstract: ***In this paper we define β -Monotone property, Soft Metric Space, b-Soft Metric Space and then we demon start some results. In the first result we prove coupled soft fixed point theorem in ordered soft b- metric space with β -monotone property. In the second theorem demonstrate coupled soft co-incidence fixed point theorem for mapping satisfying generalized contractive conditions with β –monotone property in an ordered soft b-metric space.***

Mathematical Subject Classification: 47H10, 54E50

Keywords: Soft point, soft metric space, soft contractive mapping, fixed point theorem, β -monotone property.

1. Introduction and Preliminaries

Metric fixed point theory is an necessary part of mathematics because its applications use in different areas like variational and linear inequalities, improvement and approximation theory. Ali et al. [9], Agraval et al. ([14], [15], [16]), Pathak et al. [7] and many authors (see [1], [2], [3], [8], [13]) established fixed theorems in different spaces like partially ordered metric space, Metric space, Manger space, Banach space, generalized Banach space etc. A concept of soft theory as new mathematical tool for dealing with uncertainties is discussed in 1999 by Molodtsov [6]. A soft set is a collection of approximate descriptions of an object this theory has rich potential applications. On soft set theory many structures contributed by many researchers (see [5], [10], [12]). Shabir and Naz [11] were studied about soft topological spaces. In these studies, the concept of soft point is explained by different techniques. Later a notion of soft metric space and investigated some basic properties of these spaces. Recently Wadkar et.al [4] proved fixed point results in soft metric space. In this paper we first prove soft coupled fixed point theorem in ordered soft b-metric space using monotone property for contractive condition. In the next theorem we prove the coupled soft coincidence fixed point theorem for mapping satisfying generalized contractive conditions with α –monotone property in an ordered soft b-metric space.

Definition 1.1: Let Z and H are respectively an initial inverse set and a parameter set. A soft set over Z is pair denoted by (G, H) if and only if G is a mapping from E into the set of all subsets of the set X. That is G: H → P(z), where P(z) is the power set of Z.

Definition 1.2: The intersection of two soft sets (Z , C) and (Y , E) over X is a soft set denoted by (I , H) over X and is given by (Z , C)$\cap^{\sim}$ (Y , E) = (I ,H) where H = C ∩ E and ∀ε ∈ H , I (ε) = Z (ε) ∩ Y (ε).

Definition 1.3: A soft set (Z, B) over Y is said to be a null soft set denoted by ∅

if Z p = empty, for all p in A.

Definition 1.4: For all p ∈ B , if Z (p) = Y then (Z , B) is called an absolute soft set over Y.

Definition 1.5: The difference of two soft sets (H , P) and (L, P) over Y is a soft set (G , P) over X, denoted by (h , E) \ (L, E) , is defined as G(y) = F(y) \ G(y), ∀y ∈ H .

Definition 1.6: The complement of soft set (H , B) is denoted by $(H , B)^C$ and is defined as $(H, B)^C = (H^c, B)$,where $H^c : B \to P(Y)$ is a mapping given by H^c (β) = Y − H (β), for all β .

Definition 1.7: Let R be the set of real numbers and B(R) be the collection of all nonempty bounded subsets of R and E taken as a set of parameters. Then a mapping Y:E→B(R) is called a soft real set, and it is denoted by (Y,E).

2. Main Results

Let (Y, ≤) be a partially ordered soft set and $\tilde{\rho}$ be a soft metric on Y such that $(Y, \tilde{\rho}, H)$ is a complete soft b-metric space. Consider a product $(Y, \tilde{\rho}, H)$ X $(Y, \tilde{\rho}, H)$ with the following partial order. For all $(x_\varphi, y_\delta) \in (Y, \tilde{\rho}, H)x\,(Y, \tilde{\rho}, H)$. We have $(u_\varphi, v_\delta) \geq (x_\varphi, y_\delta) \Leftrightarrow x_\varphi \geq u_\varphi,\ y_\delta \leq v_\delta$

Theorem 2.1: Let $((Y, \tilde{\rho}, H), \leq)$ be a partially ordered complete soft b- metric space and let S be a continuous mapping having the mixed monotone property such that For all $x_\varphi, y_\delta, u_\varphi, v_\delta \in (Y, \tilde{\rho}, H)$ and $\left(0, \frac{1}{s}\right)$ and $s \geq 1$,

We have

$$\tilde{\rho}\left(F(x_\varphi, y_\delta), (u_\varphi, v_\delta\right) \leq \beta \max \begin{bmatrix} \tilde{\rho}\,(x_\varphi\,, F(u_\varphi, v_\delta) + \tilde{\rho}\left(y_\delta, F(v_\delta, u_\varphi)\right) \\ +\tilde{\rho}\,(u_\varphi\,, F(x_\varphi, y_\delta) + \tilde{\rho}\left(v_\delta, F(y_\delta, x_\varphi)\right) \\ +\tilde{\rho}(y_\delta\,, \frac{F(u_\varphi, v_\delta)F(v_\delta, u_\varphi)F(x_\varphi, y_\delta)}{1+F(x_\varphi, y_\delta)F(u_\varphi, v_\delta)}) \end{bmatrix} \quad (2.1.1)$$

Then F has a coupled soft fixed point in soft b-metric space $(Y, \tilde{\rho}, H)$.

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Proof: Let two points $x_{\varphi}^{0}, y_{\delta}^{0} \in (Y, \tilde{\rho}, H) x (Y, \tilde{\rho}, H)$ and $x_{\varphi_1}^{1} = F(x_{\varphi}^{0}, y_{\delta}^{0})$; $y_{\delta_1}^{1} = F(y_{\delta}^{0}, x_{\varphi}^{0})$.

In general $y_{\delta_{n+2}}^{n+2} = F(y_{\delta_{n+1}}^{n+1}, x_{\varphi_{n+1}}^{n+1}), y_{\delta_{n+1}}^{n+1} = F(y_{\delta_n}^{n}, x_{\varphi_n}^{n})$

Similarly $x_{\varphi_{n+2}}^{n+2} = F(x_{\varphi_{n+1}}^{n+1}, y_{\delta_{n+1}}^{n+1}), x_{\varphi_{n+1}}^{n+1} = F(x_{\varphi_n}^{n}, y_{\delta_n}^{n})$

with $x_{\varphi}^{0} \le F(x_{\varphi}^{0}, y_{\delta}^{0}) = x_{\varphi_1}^{1}$ and $y_{\delta}^{0} \ge F(y_{\delta}^{0}, x_{\varphi}^{0}) = y_{\delta_1}^{1}$ (2.1.2)

by iteartive process, we have

$$x_{\varphi_2}^{2} = F\left(x_{\varphi_1}^{1}, y_{\delta_1}^{1}\right) = F\left(F(x_{\varphi}^{0}, y_{\delta}^{0}), F(y_{\delta}^{0}, x_{\varphi}^{0})\right) = F^2(x_{\varphi}^{0}, y_{\delta}^{0})$$

and $y_{\delta_2}^{2} = F\left(y_{\delta_1}^{1}, x_{\varphi_1}^{1}\right) = F\left(F(y_{\delta}^{0}, x_{\varphi}^{0}), F(x_{\varphi}^{0}, y_{\delta}^{0})\right) = F^2(y_{\delta}^{0}, x_{\varphi}^{0})$

By the mixed monotone property of F we obtain

$$x_{\varphi_2}^{2} = F^2(x_{\varphi}^{0}, y_{\delta}^{0}) = F\left(x_{\varphi_1}^{1}, y_{\delta_1}^{1}\right) \ge F(x_{\varphi}^{0}, y_{\delta}^{0}) = x_{\varphi_1}^{1}$$
And $y_{\delta_2}^{2} = F^2(y_{\delta}^{0}, x_{\varphi}^{0}) = F\left(y_{\delta_1}^{1}, x_{\varphi_1}^{1}\right) \le F(y_{\delta}^{0}, x_{\varphi}^{0}) = y_{\delta_1}^{1}$

In general we have for $n \in N$

$$x_{\varphi_{n+1}}^{n+1} = F^{n+1}(x_{\varphi}^{0}, y_{\delta}^{0}) = F\left(F^{n}(x_{\varphi}^{0}, y_{\delta}^{0}), F^{n}(y_{\delta}^{0}, x_{\varphi}^{0})\right) \quad (2.1.3)$$

and $y_{\delta_{n+1}}^{n+1} = F^{n+1}(y_{\delta}^{0}, x_{\varphi}^{0}) = F(F(y_{\delta}^{0}, x_{\varphi}^{0}), F(x_{\varphi}^{0}, y_{\delta}^{0}))$
It is obvious that
$$x_{\varphi}^{0} \le F(x_{\varphi}^{0}, y_{\delta}^{0}) = x_{\varphi_1}^{1} \le F^2(x_{\varphi}^{0}, y_{\delta}^{0}) = x_{\varphi_2}^{2} \le \cdots \le F^{n}(x_{\varphi}^{0}, y_{\delta}^{0}) = x_{\varphi_n}^{n} \le$$

$$y_{\delta}^{0} \ge F(y_{\delta}^{0}, x_{\varphi}^{0}) = y_{\delta_1}^{1} \ge F\left(y_{\delta_1}^{1}, x_{\varphi_1}^{1}\right) = y_{\delta_2}^{2} \ge \cdots \ge F^{n}(x_{\varphi}^{0}, y_{\delta}^{0}) = y_{\delta_n}^{n} \ge \ldots \quad (2.1.4)$$

Thus by mathematical induction principal we have for $n \in N$ $x_{\varphi}^{0} \le x_{\varphi_1}^{1} \le x_{\varphi_1}^{2} \le \cdots \le x_{\varphi_n}^{n} \le x_{\varphi_{n+1}}^{n+1} \ldots$ and $y_{\delta}^{0} \ge y_{\delta_1}^{1} \ge y_{\delta_2}^{2} \ge \cdots \ge y_{\delta_n}^{n} \ge y_{\delta_{n+1}}^{n+1}$

Thus we have by condition (2.1.1) that

$$\tilde{\rho}\left(x_{\varphi_{n+1}}^{n+1}, x_{\varphi_n}^{n}\right) = \tilde{\rho}\left(F\left(x_{\varphi_n}^{n}, y_{\delta_n}^{n}\right), F(x_{\varphi_{n-1}}^{n-1}, y_{\delta_{n-1}}^{n-1})\right)$$

$$\le \beta \max\left\{\begin{array}{c} \tilde{\rho}\left(x_{\varphi_n}^{n}, F\left(x_{\varphi_{n-1}}^{n-1}, y_{\delta_{n-1}}^{n-1}\right)\right) + \tilde{\rho}\left(y_{\delta_n}^{n}, F\left(y_{\delta_{n-1}}^{n-1}, x_{\varphi_{n-1}}^{n-1}\right)\right), \\ \tilde{\rho}\left(x_{\varphi_{n-1}}^{n-1}, F\left(x_{\varphi_n}^{n}, y_{\delta_n}^{n}\right)\right) + \tilde{\rho}\left(y_{\delta_{n-1}}^{n-1}, F\left(y_{\delta_n}^{n}, x_{\varphi_n}^{n}\right)\right), \\ \tilde{\rho}\left(y_{\delta_n}^{n}, \frac{F\left(x_{\varphi_{n-1}}^{n-1}, y_{\delta_{n-1}}^{n-1}\right).F\left(y_{\delta_{n-1}}^{n-1}, x_{\varphi_{n-1}}^{n-1}\right).F\left(x_{\varphi_n}^{n}, y_{\delta_n}^{n}\right)}{1 + F\left(x_{\varphi_n}^{n}, y_{\delta_n}^{n}\right).\left(x_{\varphi_{n-1}}^{n-1}, y_{\delta_{n-1}}^{n-1}\right)}\right) \end{array}\right\}$$

$$\le \beta \max\left\{\begin{array}{c} \tilde{\rho}\left(x_{\varphi_n}^{n}, x_{\varphi_n}^{n}\right) + \tilde{\rho}\left(y_{\delta_n}^{n}, y_{\delta_n}^{n}\right), \\ \tilde{\rho}\left(x_{\varphi_{n-1}}^{n-1}, x_{\varphi_{n-1}}^{n+1}\right) + \tilde{\rho}\left(y_{\delta_{n-1}}^{n-1}, y_{\delta_{n-1}}^{n+1}\right), \\ \tilde{\rho}\left(y_{\delta_n}^{n}, \frac{F\left(x_{\varphi_{n-1}}^{n-1}, y_{\delta_{n-1}}^{n-1}\right).F\left(y_{\delta_{n-1}}^{n-1}, x_{\varphi_{n-1}}^{n-1}\right).F\left(x_{\varphi_n}^{n}, y_{\delta_n}^{n}\right)}{F\left(x_{\varphi_n}^{n}, y_{\delta_n}^{n}\right).F\left(x_{\varphi_{n-1}}^{n-1}, y_{\delta_{n-1}}^{n-1}\right)}\right) \end{array}\right\}$$

$$\le \beta \max\left\{\begin{array}{c} \tilde{\rho}\left(x_{\varphi_n}^{n}, x_{\varphi_n}^{n}\right) + \tilde{\rho}\left(y_{\delta_n}^{n}, y_{\delta_n}^{n}\right), \\ \tilde{\rho}\left(x_{\varphi_{n-1}}^{n-1}, x_{\varphi_{n-1}}^{n+1}\right) + \tilde{\rho}\left(y_{\delta_{n-1}}^{n-1}, y_{\delta_{n-1}}^{n+1}\right), \\ \tilde{\rho}\left(y_{\delta_n}^{n}, y_{\delta_n}^{n}\right), \end{array}\right\}$$

$$\le \beta \max\left[\tilde{\rho}\left(x_{\varphi_{n-1}}^{n-1}, x_{\varphi_{n-1}}^{n+1}\right) + \tilde{\rho}\left(y_{\delta_{n-1}}^{n-1}, y_{\delta_{n-1}}^{n+1}\right)\right]$$

$$\tilde{\rho}\left(x_{\varphi_{n-1}}^{n+1}, x_{\varphi_{n-1}}^{n}\right) \le \beta\left[\tilde{\rho}\left(x_{\varphi_{n-1}}^{n-1}, x_{\varphi_{n-1}}^{n+1}\right) + \tilde{\rho}\left(y_{\delta_{n-1}}^{n-1}, y_{\delta_{n-1}}^{n+1}\right)\right] \quad (2.1.5)$$

Similarly since $y_{\delta_{n-1}}^{n-1} \ge y_{\delta_n}^{n}$ and $x_{\varphi_{n-1}}^{n-1} \le x_{\varphi_n}^{n}$ (2.1.6)

$$\tilde{\rho}\left(y_{\delta_{n-1}}^{n+1}, y_{\delta_n}^{n}\right) \le \beta\left[\tilde{\rho}\left(x_{\varphi_{n-1}}^{n-1}, x_{\varphi_{n-1}}^{n+1}\right) + \tilde{\rho}\left(y_{\delta_{n-1}}^{n-1}, y_{\delta_{n-1}}^{n+1}\right)\right]$$

$$\tilde{\rho}\left(x_{\varphi_{n-1}}^{n+1}, x_{\varphi_{n-1}}^{n}\right) + \tilde{\rho}\left(y_{\delta_{n-1}}^{n+1}, y_{\delta_n}^{n}\right) \le \beta\left[\tilde{\rho}\left(x_{\varphi_{n-1}}^{n-1}, x_{\varphi_{n-1}}^{n+1}\right) + \tilde{\rho}\left(y_{\delta_{n-1}}^{n-1}, y_{\delta_{n-1}}^{n+1}\right)\right] + \beta\left[\tilde{\rho}\left(x_{\varphi_{n-1}}^{n-1}, x_{\varphi_{n-1}}^{n+1}\right) + \tilde{\rho}\left(y_{\delta_{n-1}}^{n-1}, y_{\delta_{n-1}}^{n+1}\right)\right]$$

$$\le 2\beta\left[s\left\{\tilde{\rho}\left(x_{\varphi_{n-1}}^{n-1}, x_{\varphi_n}^{n}\right) + \tilde{\rho}\left(x_{\varphi_n}^{n}, x_{\varphi_{n-1}}^{n+1}\right)\right\} + s\left\{\tilde{\rho}\left(y_{\delta_{n-1}}^{n-1}, y_{\delta_n}^{n}\right) + \tilde{\rho}\left(y_{\delta_n}^{n}, y_{\delta_{n-1}}^{n+1}\right)\right\}\right]$$

$$2\beta s\left[\tilde{\rho}\left(x_{\varphi_{n-1}}^{n-1}, x_{\varphi_n}^{n}\right) + \tilde{\rho}\left(y_{\delta_{n-1}}^{n-1}, y_{\delta_n}^{n}\right)\right] + 2\,\beta s\left[\tilde{\rho}\left(x_{\varphi_n}^{n}, x_{\varphi_{n-1}}^{n+1}\right) + \tilde{\rho}\left(y_{\delta_n}^{n}, y_{\delta_{n-1}}^{n+1}\right)\right]$$

(1-
$2\beta s)\left[\tilde{\rho}\left(x_{\varphi_n}^{n}, x_{\varphi_{n-1}}^{n+1}\right) + \tilde{\rho}\left(y_{\delta_n}^{n}, y_{\delta_{n-1}}^{n+1}\right)\right] \le 2\beta s\left[\tilde{\rho}\left(x_{\varphi_{n-1}}^{n-1}, x_{\varphi_n}^{n}\right) +$
$\rho y\delta n-1n-1, y\delta nn$

$$\left[\tilde{\rho}\left(x_{\varphi_n}^{n}, x_{\varphi_{n-1}}^{n+1}\right) + \tilde{\rho}\left(y_{\delta_n}^{n}, y_{\delta_{n-1}}^{n+1}\right)\right] \le \frac{2\beta s}{(1-2\beta s)}\left[\tilde{\rho}\left(x_{\varphi_{n-1}}^{n-1}, x_{\varphi_n}^{n}\right) + \tilde{\rho}\left(y_{\delta_{n-1}}^{n-1}, y_{\delta_n}^{n}\right)\right]$$

Let suppose that $h = \frac{2\beta s}{(1-2\beta s)}$ and $d_m = \tilde{\rho}\left(x_{\varphi_n}^{n}, x_{\varphi_{n-1}}^{n+1}\right) + \tilde{\rho}\left(y_{\delta_n}^{n}, y_{\delta_{n-1}}^{n+1}\right)$ Then $d_m \le h d_{m-1}$

Similarly it can be proved that $d_{m-1} \le h d_{m-2}$.

Therefore $d_m \le h d_{m-1} \le h^2 d_{m-2} \le \cdots h^n d_0$. (2.1.7)

This implies that $\lim_{n\to\infty} d_n = 0$.
Thus $\lim_{n\to\infty} d\left(x_{\varphi_{n-1}}^{n+1}, x_{\varphi_n}^{n}\right) = \lim_{n\to\infty} d\left(y_{\delta_{n-1}}^{n+1}, y_{\delta_n}^{n}\right) = 0$

For each $p \ge n$, by (2.1.7) and repeat the application of triangle inequality that we optain that

$$\left(y_{\delta_n}^{n}, y_{\delta_p}^{p}\right) \le s\tilde{\rho}\left(y_{\varphi_n}^{n}, y_{\varphi_{n-1}}^{n+1}\right) + s^2\tilde{\rho}\left(y_{\varphi_{n-1}}^{n+1}, y_{\varphi_{n+2}}^{n+2}\right) + s^3\tilde{\rho}\left(y_{\varphi_{n-2}}^{n+2}, y_{\varphi_{n-1}}^{n+1}\right) + \cdots s^m\tilde{\rho}\left(y_{\varphi_n}^{n}, y_{\varphi_{n-1}}^{n+1}\right)$$

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And
$\left(x_{\delta_n}^n, x_{\delta_p}^p\right) \le s\tilde{\rho}\left(x_{\varphi_n}^n, x_{\varphi_{n-1}}^{n+1}\right) + s^2\tilde{\rho}\left(x_{\varphi_{n-1}}^{n+1}, x_{\varphi_{n+2}}^{n+2}\right) + s^3\tilde{\rho}\left(x_{\varphi_{n-2}}^{n+2}, x_{\varphi_{n-1}}^{n+1}\right) + \cdots s^m\tilde{\rho}\left(x_{\varphi_n}^n, x_{\varphi_{n-1}}^{n+1}\right)$
Adding these we get
$\left(y_{\delta_n}^n, y_{\delta_p}^p\right) + \left(x_{\delta_n}^n, x_{\delta_p}^p\right) \le s\left[\tilde{\rho}\left(y_{\varphi_n}^n, y_{\varphi_{n-1}}^{n+1}\right) + \tilde{\rho}\left(x_{\varphi_n}^n, x_{\varphi_{n-1}}^{n+1}\right)\right] + s^2[\tilde{\rho}\left(x_{\varphi_{n-1}}^{n+1}, x_{\varphi_{n+2}}^{n+2}\right) + \tilde{\rho}\left(x_{\varphi_{n-1}}^{n+1}, x_{\varphi_{n+2}}^{n+2}\right) + s^3\left[\tilde{\rho}\left(x_{\varphi_{n-2}}^{n+2}, x_{\varphi_{n-1}}^{n+1}\right) + \tilde{\rho}\left(y_{\varphi_{n-2}}^{n+2}, y_{\varphi_{n-1}}^{n+1}\right)\right] \ldots s^m\left[\tilde{\rho}\left(x_{\varphi_n}^n, x_{\varphi_{n-1}}^{n+1}\right) + \tilde{\rho}\left(y_{\varphi_{n-2}}^{n+2}, y_{\varphi_{n-1}}^{n+1}\right)\right]$
$\le s[h^n+h^{n+1}+h^{n+2} + h^{n+3} + h^{n+4} \ldots\ldots\ldots h^{m-1}]d_0 \le s\frac{h^n}{1-h^n}$
$d_0 \to 0$ as $n \to \infty$

There $\left\{x_{\varphi_n}^n\right\}$ and $\left\{y_{\mu_n}^n\right\}$ are chouchy sequences.Since(X, $\tilde{\rho}$, E)is complete b- soft metric space. There exist y_μ, x_φ in (X, $\tilde{\rho}$, E) such that $\lim_{n\to\infty} x_{\varphi_n}^n = x_\varphi$

And $\lim_{n\to\infty} y_{\mu_n}^n = y_\mu$. Thus by taking limit $n \to \infty$, in equation we get
$x_\varphi = \lim_{n\to\infty} x_{\varphi_n}^n =$ $\lim_{n\to\infty} S[x_{\varphi_{n-1}}^{n-1}, y_{\delta_{n-1}}^{n-1}] = S$
$\lim_{n\to\infty}[x_{\varphi_{n-1}}^{n-1}, y_{\delta_{n-1}}^{n-1}] =$
$S(x_\varphi, y_\delta)$ and
$y_\delta = \lim_{n\to\infty} y_{\delta_n}^n$
$= \lim_{n\to\infty} S[y_{\delta_{n-1}}^{n-1}, x_{\delta_{n-1}}^{n-1}] = \lim_{n\to\infty}[x_{\delta_{n-1}}^{n-1}, y_{\delta_{n-1}}^{n-1}] = S(x_\varphi, y_\delta)$

Therefore $x_\varphi = S(x_\varphi, y_\delta)$and $y_\delta = S(y_\delta, x_\varphi)$.Thus S has coupled soft fixed point in (X, $\tilde{\rho}$,E).

In next theorem we prove coupled soft coincidence fixed point theorem for mapping satisfying generalized contractive conditions with –monotone in an ordered soft b-metric space.

Theorem2.2: Let ((X,$\tilde{\rho}$, E),≤) be a partially ordered set and $\tilde{\rho}$: (X, $\tilde{\rho}$,E) x (X, $\tilde{\rho}$,E)→ R be a soft b-mtric defined on X with coefficient $s \ge 1$. Let β: (X, $\tilde{\rho}$,E)→ (X, $\tilde{\rho}$, E) and
S: (X, $\tilde{\rho}$,E)→ (X, $\tilde{\rho}$, E)be two mappings such that
$\tilde{\rho}(S(x_\varphi, y_\delta).S(u_\varphi, v_\delta)) + \tilde{\rho}(S(y_\delta, x_\varphi).S(v_\delta, u_\varphi)) \le k\left\{\tilde{\rho}\left(\beta x_\varphi, \beta u_\varphi\right) + \tilde{\rho}(\beta y_\delta, \beta v_\delta)\right\}$; (2.2.1)
for some $k \in \left[0, \frac{1}{s}\right)$ and for all $x_\varphi, y_\delta\ u_\varphi, v_\delta \in$(X, $\tilde{\rho}$, E) with $\beta x_\varphi \ge \beta u_\varphi$ and $\beta y_\delta \ge \beta v_\delta$.We further assume the following hypothesis:
1) $S(X \times X) \subseteq \beta(X)$
2) $\beta(X)$ is complete
3) β is continuous and commute with S.
4) S has the mixed β-monotone property on X and eighter S is continuous or X has the following property
   a) If a non decreasing sequence and $\left\{x_{\varphi_n}^n\right\} \to x_\varphi$ then $\left\{x_{\varphi_n}^n\right\} \le x_\varphi$.
   b) If a non increasing sequence and $\left\{y_{\delta_n}^n\right\} \to y_\delta$ then $\left\{y_{\delta_n}^n\right\} \le y_\delta$.

If there exist two elements $x_{\varphi_0}^0, y_{\delta_0}^0$ in (X, $\tilde{\rho}$,E) with $\beta x_{\varphi_0}^0 \le S(x_{\varphi_0}^0, y_{\delta_0}^0)$
$\beta y_{\delta_0}^0 \ge S(y_{\delta_0}^0, x_{\varphi_0}^0)$,
Then S and β have unique a coupled soft fixed point that is there exist a unique $x_\varphi \in (X, \tilde{\rho}, E)$ such that $x_\varphi = S(x_\varphi, x_\varphi) = \beta x_\varphi$.

Proof: Let$x_{\varphi_0}, y_{\delta_0} \in (X, \tilde{\rho}, E)$ be such that $\beta x_{\varphi_0} \le S\left(x_{\varphi_0}, y_{\delta_0}\right)$ and $\beta y_{\delta_0} \ge S\left(y_{\delta_0}, x_{\varphi_0}\right)$.Since S(X x X) ≥ β(X), We can choose $x_{\varphi_0}^1, y_{\delta_0}^1 \in$ (X, $\tilde{\rho}$,E) such that $\beta x_{\varphi_0}^1 = S(x_{\varphi_0}, y_{\delta_0})$ and $\beta y_{\delta_0}^1 = S(y_{\delta_0}, x_{\varphi_0})$.Again since S(X x X) ≥ β(X),we can choose $x_{\varphi_0}^2, y_{\delta_0}^2 \in$ (X, $\tilde{\rho}$,E) such that $\beta x_{\delta_0}^2 = S(x_{\varphi_0}^1, y_{\delta_0}^1)$ and $\beta y_{\delta_0}^2 = S(y_{\delta_0}^1, x_{\varphi_0}^1)$.

Continuing this process we can constract two sequences $\left\{x_{\varphi_n}^n\right\}$and $\left\{y_{\delta_n}^n\right\}$ in X.
Such that $\beta x_{\varphi_{n+1}}^{n+1} = S(x_{\varphi_0}^n, y_{\delta_0}^n)$ (2.2.2)
And $\beta y_{\delta_{n+1}}^{n+1} = S(y_{\delta_0}^n, x_{\varphi_0}^n)$ for all n (2.2.3)

Now we will prove that for all $n \ge 0$.
$\beta x_{\varphi_n}^n \le \beta x_{\varphi_{n+1}}^{n+1}$, (2.2.4)
$\beta y_{\delta_n}^n \ge \beta y_{\delta_{n+1}}^{n+1}$ (2.2.5)

We shall use the mathematical law of induction. Let n = 0.
Since $\beta x_{\varphi_0} \le S\left(x_{\varphi_0}, y_{\delta_0}\right)$ and $\beta y_{\delta_0} \ge S\left(y_{\delta_0}, x_{\varphi_0}\right)$ and $\beta x_{\varphi_0}^1 = S(x_{\varphi_0}, y_{\delta_0})$ and $\beta y_{\delta_0}^1 = S(y_{\delta_0}, x_{\varphi_0})$ we have $\beta x_{\varphi_0} \le x_{\varphi_0}^1$ and and $\beta y_{\delta_0} \ge \beta y_{\delta_0}^1$

That is (2.2.4) and (2.2.5) holds for all n = 0. We assume that (2.2.4) and (2.2.5) holds for n> *0*

As S has the mixed monotone property and $\beta x_{\varphi_n}^n \le \beta x_{\varphi_{n+1}}^{n+1}$ and
$\beta y_{\delta_n}^n \ge \beta y_{\delta_{n+1}}^{n+1}$,we get

$\beta x_{\varphi_{n+1}}^{n+1} = S\left(x_{\varphi_n}^n, y_{\delta_n}^n\right) \le S(x_{\varphi_{n+1}}^{n+1}, y_{\delta_n}^n)$ and
$S(y_{\delta_{n+1}}^{n+1}, x_{\varphi_n}^n) \le S\left(y_{\delta_n}^n, x_{\varphi_n}^n\right) = \beta y_{\delta_{n+1}}^{n+1}$

Also for the same reason we have
$\beta x_{\varphi_{n+1}}^{n+2} = S\left(x_{\varphi_{n+1}}^{n+1}, y_{\delta_{n+1}}^{n+1}\right) \ge S(x_{\varphi_{n+1}}^{n+1}, y_{\delta_n}^n)$ and $(y_{\delta_{n+1}}^{n+1}, x_{\varphi_n}^n) \le S\left(y_{\delta_n}^n, x_{\varphi_n}^{n+1}\right) = \beta y_{\delta_{n+1}}^{n+1}$

From (2.2.2) and (2.2.3) we obtain $\beta x_{\varphi_{n+1}}^{n+1} \le \beta x_{\varphi_{n+1}}^{n+2}$ and $\beta y_{\delta_{n+1}}^{n+1} \ge \beta y_{\delta_{n+2}}^{n+2}$. Thus by mathematical induction we conclude that (2.2.4) and (2.2.5) holds for all n≥ 0. Continuing this process one can easily verify that
$\beta x_{\varphi_0} \le \beta x_{\varphi_1}^1 \le \beta x_{\varphi_2}^2 \le \beta x_{\varphi_3}^3 \le \cdots . \beta x_{\varphi_n}^n \le \beta x_{\varphi_{n+1}}^{n+1} \le \cdots \ldots$ (2.2.6)

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$\beta y_{\delta_0} \geq \beta y^1_{\delta_1} \geq \beta y^2_{\delta_2} \geq \beta y^3_{\delta_3} \ldots\ldots \geq \beta y^n_{\delta_n} \geq \beta y^{n+1}_{\delta_{n+1}} \geq \cdots\ldots$ (2.2.7)

Now if $(x^{n+1}_{\varphi_{n+1}}, y^{n+1}_{\delta_{n+1}}) = (x^n_{\varphi_n}, y^n_{\delta_n})$,then S and β have coupled soft coincidence point.
So assume $(x^{n+1}_{\varphi_{n+1}}, y^{n+1}_{\delta_{n+1}}) \neq (x^n_{\varphi_n}, y^n_{\delta_n})$ for all $n \geq 0$.i.e. we assume that either
$\beta x^{n+1}_{\varphi_{n+1}} = (x^n_{\varphi_n}, y^n_{\delta_n}) \neq \beta x^n_{\varphi_n}$ or $\beta y^{n+1}_{\delta_{n+1}} = S\left(y^n_{\delta_n}, x^n_{\varphi_n}\right) \neq \beta y^n_{\delta_n}$.

Again let p and n be two positive integer such that $p > n$ then we can write
$\tilde{\rho}\left(\beta x^n_{\varphi_n}, \beta x^p_{\varphi_p}\right) \leq$
$s\left\{\tilde{\rho}(\beta x^n_{\varphi_n}, \beta x^{n+1}_{\varphi_{n+1}})\right\} + s\left\{\tilde{\rho}(\beta x^{n+1}_{\varphi_{n+1}}, \beta x^p_{\varphi_p})\right\} \leq$
$s\left\{\tilde{\rho}(\beta x^n_{\varphi_n}, \beta x^{n+1}_{\varphi_{n+1}})\right\} + s^2\left\{\tilde{\rho}(\beta x^{n+1}_{\varphi_{n+1}}, \beta x^{n+2}_{\varphi_{n+2}})\right\} +$
$s^3\left\{\tilde{\rho}(\beta x^{n+2}_{\varphi_{n+2}}, \beta x^{n+3}_{\varphi_{n+3}})\right\} + \cdots . s^{p-n-1}\left\{\tilde{\rho}(\beta x^{p+1}_{\varphi_{p+1}}, \beta x^n_{\varphi_n})\right\}.$
Similarly
$\tilde{\rho}\left(\beta y^n_{\delta_n}, \beta y^p_{\delta_p}\right) \leq$
$\tilde{\rho}\left\{s(\beta y^n_{\delta_n}, \beta y^{n+1}_{\delta_{n+1}})\right\} + s^2\left\{(\tilde{\rho}(\beta y^{n+1}_{\delta_{n+1}}, \beta y^{n+2}_{\delta_{n+2}})\right\} +$
$\cdots s^{p-n-1}\left\{(\tilde{\rho}(\beta y^{p-1}_{\delta_{p-1}}, \beta y^p_{\delta_p})\right\}.$

Therefore

$$\tilde{\rho}\left(\beta x^n_{\varphi_n}, \beta x^p_{\varphi_p}\right) + \tilde{\rho}\left(\beta y^n_{\delta_n}, \beta y^p_{\delta_p}\right) \leq sd_n + s^2 d_{n+1} + \cdots . + s^{p-n-1} d_{p-1}$$
$$\leq sk^n d_0 + s^2 k^{n+1} d_0 + \cdots . + s^{p-n-1} k^n d_0$$
$$\leq sk^n d_0 \frac{1}{1-sk} \to 0 \text{ ; as } n \to \infty$$

Hence $\left\{\beta x^n_{\varphi_n}\right\}$ and $\left\{\beta y^n_{\delta_n}\right\}$ are two Cauchy sequences in βX and βX is complete.

Thus there exist two soft point say x_φ, y_δ in X such that $\left\{\beta x^n_{\varphi_n}\right\} = \beta x_\varphi = \Omega$
and
that $\left\{\beta x^n_{\varphi_n}\right\} \to \beta x_\varphi = \xi$ as $n \to \infty$. Hence S is complete and so
$\beta\left\{\beta x^{n+1}_{\varphi_{n+1}}\right\} = \beta\left(S(\beta x^n_{\varphi_n}, \beta y^n_{\delta_n})\right) = S(\beta x^n_{\varphi_n}, \beta y^n_{\delta_n})$ (Since S andβ are commutative)
$\beta(\Omega) = S(\Omega, \xi)$ (Since S andβ are continuous)

Similarly we can show that $\beta(\Omega) = S(\Omega, \xi)$ .Thus $S(\Omega, \xi)$ is point of coincidence for S and β.Again le we getthat holds,by (2.2.6) $\left\{\beta x^n_{\varphi_n}\right\}$ is a non decreasing sequence and $\beta x^n_{\varphi_n} \to \Omega$, therefore $\beta x^n_{\varphi_n} \leq \Omega$ for all n.Similarly by (2.2.7) we get that $\left\{\beta y^n_{\delta_n}\right\}$ is non increasing sequence and
$\beta y^n_{\delta_n} \to \xi$, so $\beta y^n_{\delta_n} \geq \xi$ for all n,
then
$\tilde{\rho}\left(\beta(\Omega), S(\Omega, \xi)\right) \leq s\tilde{\rho}$
$s\left(\beta(\Omega), \beta\beta x^{n+1}_{\varphi_{n+1}}\right) + s\tilde{\rho}\left(\beta\beta x^{n+1}_{\varphi_{n+1}}, S(\Omega, \xi)\right)$
$s\left(\beta(\Omega), \beta\beta x^{n+1}_{\varphi_{n+1}}\right) + s\tilde{\rho}\left(\beta(S(x^n_{\varphi_n}, y^n_{\delta_n})), S(\Omega, \xi)\right)$
$s\left(\beta(\Omega), \beta\beta x^{n+1}_{\varphi_{n+1}}\right) + s\tilde{\rho}\left(\beta(S(\beta x^n_{\varphi_n}, \beta y^n_{\delta_n})), S(\Omega, \xi)\right)$
$\leq \left(\beta(\Omega), \beta\beta x^{n+1}_{\varphi_{n+1}}\right) + s\tilde{\rho}\left(\beta(S(\beta x^n_{\varphi_n}, \beta y^n_{\delta_n})), S(\Omega, \xi)\right)$
$+ s\tilde{\rho}\left((S(\beta y^n_{\delta_n}, \beta x^n_{\varphi_n})), S(\Omega, \xi)\right)$
$\leq \left(\beta(\Omega), \beta\beta x^{n+1}_{\varphi_{n+1}}\right) + sk\left\{\beta\left(\beta\beta x^n_{\varphi_n}, \beta\Omega\right) + s\tilde{\rho}\left(\beta\beta y^n_{\delta_n}, \beta\xi\right)\right\}$
(2.2.8)
Since β is continuous ,$\beta\beta x^n_{\varphi_n} \to \beta\Omega$ and $\beta\beta y^n_{\delta_n} \to \beta\xi$ and hence the right hand side of equation (2.2.8) becomes zero as $n \to \infty$.

Thus $\beta(\Omega) = S(\Omega, \xi)$, similarly we can show that
$\beta(\xi) = S(\xi, \Omega)$.
$\tilde{\rho}(\beta(\Omega), S(\Omega, \xi))$
$\tilde{\rho}\left(\beta(\Omega), S(\Omega, \xi)\right) + \tilde{\rho}(\beta\xi, \Omega)$
$= \tilde{\rho}\left(S(\Omega, \xi), S(\xi, \Omega)\right)$
$+ \tilde{\rho}(S(\xi, \Omega), S(\Omega, \xi)$
$\leq k\{\tilde{\rho}\{(\beta\Omega, \beta\xi) + \tilde{\rho}\{(\beta\xi, \beta\Omega)\}$
$2\tilde{\rho}\{(\beta\Omega, \beta\xi) \leq 2k\tilde{\rho}\{(\beta\Omega, \beta\xi) \to \tilde{\rho}\{(\beta\Omega, \beta\xi) \leq k\tilde{\rho}\{(\beta\Omega, \beta\xi)$

Since $k < \frac{1}{s}$;
$\tilde{\rho}\{(\beta\Omega, \beta\xi) = 0$. Thus $\beta\Omega = \beta\xi$, Hence $S(\Omega, \xi) = S(\Omega) = \beta(\xi) = S(\xi, \Omega)$

Finally
$\tilde{\rho}(\Omega, \beta\Omega) \leq s\tilde{\rho}\left(\Omega, \beta x^{n+1}_{\varphi_{n+1}}\right) + s\tilde{\rho}\left(\beta x^{n+1}_{\varphi_{n+1}}, \beta\Omega\right) +$
$s\tilde{\rho}\left(\Omega, \beta x^{n+1}_{\varphi_{n+1}}\right) + s\tilde{\rho}\left(S\left(x^n_{\varphi_n}, y^n_{\delta_n}\right), S(\Omega, \xi)\right)$

In the same way
$$\tilde{\rho}(\xi, \beta\xi) \leq s\tilde{\rho}\left(\xi, \beta y^{n+1}_{\varphi_{n+1}}\right) + s\tilde{\rho}\left(\beta y^{n+1}_{\varphi_{n+1}}, \beta\xi\right)$$
$$\leq s\tilde{\rho}\left(\xi, \beta y^{n+1}_{\varphi_{n+1}}\right) + s\tilde{\rho}\left(S\left(y^n_{\delta_n}, x^n_{\varphi_n}\right), S(\xi, \Omega)\right)$$

Therefore
$(1-ks^2)[\tilde{\rho}(\Omega, \beta\Omega) + \tilde{\rho}(\xi, \beta\xi)] \leq s\left\{\tilde{\rho}\left(\Omega, \beta x^{n+1}_{\varphi_{n+1}}\right) +$
$\rho\xi,\beta y\varphi n+1 n+1 +$
$$s^2\left\{\left(\beta y^n_{\varphi_n}, \Omega\right) + \left(\beta x^n_{\varphi_n}, \xi\right)\right\} \to 0 \text{ as } \to \infty$$

Thus $\tilde{\rho}(\Omega, \beta\Omega) = 0 = \tilde{\rho}(\xi, \beta\xi)$
$\xi = \beta\xi$ and $\Omega = \beta\Omega$
$\beta y_\delta = S(y_\delta, y_\delta) = y_\delta$
This means that S and β have a common soft fixed point.

3. Conclusion

In this paper the investigations concerning the existence of coupled soft fixed point theorem for a contractive condition with monotone property and $\beta -$ monotony property in an ordered soft b- metric space are established.

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References

[1] B. E. Rhoades, A comparison of various definitions of contractive mappings, Trans. Amer. Math. Soc., **226** (1977), 257-290. https://doi.org/10.1090/s0002-9947-1977-0433430-4
[2] B. R. Wadkar, B. K. Singh, R. Bhardwaj and A. Tiwari, Coupled fixed point theorem in partially ordered metric space, Mathematical Theory and Modeling, **3** (2013), no. 6, 363-371.
[3] R. Bhardwaj, B. R. Wadkar, Basant Singh, Fixed point theorems in Generalized banach Space, *International Journal of Computer and Mathematical Sciences*, **4** (2015), 96-102.
[4] B. R. Wadkar, R. Bhardwaj,V. N. Mishra, B. Sing, Fixed point Results Related to Soft sets, *Australian Journal of Basic and Applied Sciences*, **10** (2016), no. 16, 128-137.
[5] C. Gunduz (Aras), A. Sonmez, H. Çakallı, On Soft Mappings, arXiv:1305. 454 5v1 [math.GM], 16 May 2013.
[6] D. Molodtsov, Soft set-theory-first results, *Comput. Math. Appl.*, **37** (1999), 19-31. https://doi.org/10.1016/s0898-1221(99)00056-5
[7] H. K. Pathak, Ravi P. Agarwal, Yeol Je Cho, Coincidence and fixed points for multi-valued mappings and its application to nonconvex integral inclusions, *Journal of Computational and Applied Mathematics*, **283** (2015), 201–221. https://doi.org/10.1016/j.cam.2014.12.019
[8] H. Long-Guang, Z. Xian, Cone metric spaces and fixed point theorems of contractive mappings, *J. Math. Anal. Appl.*, **332** (2007), 1468-1476. https://doi.org/10.1016/j.jmaa.2005.03.087
[9] J. Ali, M. Imdad, D Mihet and M. Tanveer, Common fixed points of Strict contractions in Menger Spaces, *Acta Math. Hungar.*, **132** (2011), no. 4, 367-386. https://doi.org/10.1007/s10474-011-0105-3
[10] M. I. Ali, F. Feng, X. Liu, W. K. Min and M. Shabir, On some new operations in soft set theory, *Comput. Math. Appl.*, **57** (2009), 1547-1553. https://doi.org/10.1016/j.camwa.2008.11.009
[11] M. Shabir and M. Naz, On soft topological spaces, *Comput. Math. Appl.*, **61** (2011), 1786-1799. https://doi.org/10.1016/j.camwa.2011.02.006
[12] P. K. Maji, R. Biswas, A. R. Roy, Soft set theory, *Comput. Math. Appl.*, **45** (2003), 555-562. https://doi.org/10.1016/s0898-1221(03)00016-6
[13] P. K. Mishra, S. Sweta, S. K. Banerjee, Some fixed point theorems in b- metric space, *Turkish J. Anal. Number Theory*, **2** (2014), no. 1, 19-22. https://doi.org/10.12691/tjant-2-1-5
[14] R. Agraval, E. Karapinar and B. Samet, An essential Remark on fixed point results on multiplicative metric spaces, *Fixed Point Theory and Applications*, **2016** (2016). https://doi.org/10.1186/s13663-016-0506-7
[15] R. Agraval, E. Karapinar, A. Francisco R. Hierro, On an extension of contractivity conditions via auxiliary functions, *Fixed Point Theory and Applications*, **2015** (2015), 104. https://doi.org/10.1186/s13663-015-0354-x
[16] R. P. Agarwal, M. A. Alghamdi, D. O'Regan and N. Shahzad, Fixed Point Theory for Cyclic Weak Kannan Type Mappings, *Journal of the Indian Math. Soc.*, **82** (2015), 11–21.
[17] S. Das and S. K. Samanta, Soft real sets, soft real numbers and their properties, *J. Fuzzy Math.*, **20** (2012), no. 3, 551-576.
[18] S. Das and S. K. Samanta, On soft metric spaces, *J. Fuzzy Math.*, **21** (2013), 707-734.

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Implementation of Common Soft Fixed Point with Intuitionistic Soft Fuzzy Metric Space

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Abstract: In this article, we define compatible mappings of type (K) in intuitionistic fuzzy metric space and prove a general Soft fixed point theorem with an application to self mappings on a full intuitionistic fuzzy metric space. Our conclusion broadens and enhances earlier, related findings in the literature.
Keywords: Intuitionistic fuzzy metric space, weakly compatible, compatible mappings of type (K),Soft set,soft fixed point

Introduction

Functional analysis must include the examination of soft fixed points. Studying frequently occurring soft fixed points of mappings that satisfy contractive type constraints has proven to be a very fruitful field of research. The idea of a fuzzy set was created in 1965 by L.A. Zade[11]. Following that, fuzzy metric spaces were introduced by Kramosil and Michalek [13]. George and Veeramani [1] used continuous t-norms to create the idea of fuzzy metric spaces. In 1986, G. Jungck [5] was the first to put forth the concept of compatible mappings. By G. Jungck, P. P. Murthy, and Y. J. Cho in 1993 [6,]. Compatible mappings of type (P) were introduced by M. Kang

[7] and contrasted with compatible mappings of type (A) and compatible mappings. In 1998,. The idea of compatible mappings of type (K) in metric space was recently proposed by Jha et al. [10], who also demonstrated that each compatible mapping of type (K) is independent of the others. A common soft fixed point theorem for compatible mappings of type (K) in intuitionistic fuzzy metric spaces is established in this study with an accompanying illustration. Molodtsov [18] introduced the idea of soft sets as a novel mathematical approach for problem-solving. A parameterized family of subsets of the full set is what Molodtsov[1] refers to as a soft set. Each element of the soft set is thought of as a collection of nearby items, and the soft set is a parameterized family of subsets of the full set. Numerous scholars have been studying the foundations of soft set theory during the past few years. A OR was presented by Maji et al. [19]. They conducted study and discussed the fundamental characteristics of their methods. Pei and Miao [20] examined the connection between information systems and soft sets and redefined the terms intersection and subset of soft sets. [21] with a focus on their fundamental features. Soft matrix theory was introduced by Cagman and Enginoglu [22]

Preliminaries

Definition 2.1. [12] Any non empty set Y and B is a fuzzy set in Y is a mapping with domain Y and [0,1].

Definition 2.2. [1] A binary operation $\diamond : [0, 1] \times [0, 1] \to [0, 1]$ is a continuous t-norm if $\diamond$ is **satisfying the following conditions:**

1. $\diamond$ is commutative and associative;
2. $\diamond$ is continuous;
3. $p \diamond 1 = p$ for all $a \in [0, 1]$;
4. $p \diamond q \leq r \diamond s$ whenever $p \leq r$ and $q \leq s$, and $p, q, r, s \in [0, 1]$.

Definition 2.3. [2] A binary operation $* :[0, 1] \times [0, 1] \to [0, 1]$ is a continuous t-conorm if it satisfies the following conditions:

1. is commutative and associative;
2. *is continuous;
3. $p *0 = p$ for all $p\in [0, 1]$;
4. $p * q \leq r * s$ whenever $p \leq r$ and $q \leq s$, for each $p, q, r, s \in [0, 1]$.

Definition 2.4. [3] Let (Y,P , Q, *, $\diamond$) be an intuitionistic fuzzy metric space.

1. A sequence {xn} in Y is called cauchy sequence if for each $t > 0$ and $P > 0$, $\lim \omega\to\infty P (y_{\omega+p},, t) = 1$and $\lim \omega\to\infty Q (y_{\omega+p},y_{\omega}, t) = 0$.
2. A sequence {yn} in Y is convergent to $y \in Y$ if $\lim \omega\to\infty Q (y_{\omega+p},, t) = 1$ and $\lim \omega\to\infty Q (y_{\omega+p},y_{\omega}, t) = 0$.
3. for each $t > 0$.
4. An intuitionistic fuzzy metric space is said to be complete if every Cauchy sequence is convergent.

Definition 2.5.[10] The self mappings B and R of a fuzzy metric space (Y, P, $*$) are said to be compatible of type (E) iff $\lim\omega \to\infty P(BBy_{\omega}, BRy_{\omega}, t) = 1$, $\lim\omega \to\infty P(BBy_{\omega}, Ry, t) = 1$,
$lim_{\omega\to\infty}$ $P(BRy_{\omega}, Ry, t) = 1$ and $lim_{\omega\to\infty}$ $P(RRy_{\omega}, RBy_{\omega}, t) = 1$,$lim_{\omega\to\infty} P(RRy_{\omega}, By, t) = 1$,
$lim_{\omega\to\infty} P(RRy_{\omega}, By, t) = 1$.

Definition 2.6.[10] The self mappings B and R of a metric space (Y, d) are said to be compatibleof type (K) iff $lim_{\omega\to\infty}$ BBy_ω = Ry and $lim_{\omega\to\infty}$RRy_ω = By, whenever {$y\omega$} is a sequence in Y such that $lim_{\omega\to\infty}$ By_ω = $lim_{\omega\to\infty}$Ry_ω = y for some y in Y.

Definition 2.7 The self mappings B and R of a intuitionistic fuzzy metric space (Y, P, Q, *, ⋄) are said to be compatible of type (K) iff $lim_{\omega\to\infty}$ P(BBy_ω, Ry, t) = 1 and $lim_{\omega\to\infty}$ P(RRy_ω, By, t) = 1, whenever {$y\omega$} is a sequence in Y such that $lim_{\omega\to\infty}$ By_ω =$lim_{\omega\to\infty}$ Ry_ω = y for some x in Y and t
> 0.

Definition 2.10:[18]Let(Y,d) is a metric space and the set (F, B) is a soft set of Y is called null soft set and it is denoted by φ.iffor all ε ∈ B , F(ε) = ϕ. (empty set)

Definition 2.11:[19]Let (Y,d) is a metric space and suppose that the set (F, B) is a soft set over (Y, d) is called absolute soft set. If for all ε ∈ B, F(ε) = Y.

Definition 2.12:**[20]**Let (Y,d) is a metric space and if the difference (F,A) of two soft sets (F,A) and (F,A) over Y is denoted by (F,A)/(F,A)and is defined as
F(e) = H(e)\G(e) for all e ∈ A.

Lemma 2.14. [16] Let (Y, P, Q, *,⋄) be an intuitionistic fuzzy metric space. If there exists a constant k ∈ (0, 1) such that, P($y\omega$+2, $y\omega$+1 kt) ≥ P($y\omega$+1, $y\omega$, t), Q ($y\omega$+1, $y\omega$ + 1,kt) ≤ Q($y\omega$+1, $y\omega$
, t) for every t > 0 and n =1, 2,... then {$y\omega$} is a Cauchy sequence in Y .

Definition 2.13:[21]For any two soft real numbers
1. $\tilde{p} \leq \tilde{q}$, if $\tilde{p}$ (e) ≤ $\tilde{q}$(e), for all e ∈ A.
2. $\tilde{p} \geq \tilde{q}$ if $\tilde{p}$ (e) ≥ $\tilde{q}$ (e), for all e ∈ A.
3. $\tilde{p} < \tilde{q}$ if $\tilde{p}$ (e) < $\tilde{q}$ (e), for all e ∈ A.
4. $\tilde{p} > \tilde{q}$, if $\tilde{p}$ (e) > $\tilde{q}$ (e), for all e ∈ A.

Definition 2.14:[22]The complement of a soft set (P,B) is denoted by (P, B)^c and is defined by
(P, B)^c= (P^c, B) whereF : B→ Q(Y) is mapping given by (β) = Y− P(β) ,∀ β∈ B.

Definition 2.15:[23] Let R be the set of real numbers and C(R)be the collection of all nonempty Bounded subsets of R and C taken as a set of parameters. Then a mapping
A:H→B(R) is said to be a soft real set. It is denoted by (A,H).If specifically (A,H) isa singleton soft set, then identifying (A,H) with the corresponding soft element, it will be said to a soft real number and denoted ã, betc.
Let 0,1 are any two soft real numbers where 0(e) = 0,1 (e) = 1 for all e ∈ H, respectively

Main Results

If the following requirements are met and B, K, L, and T are self mapping on a full intuitionistic soft fuzzy metric space (Y, P, Q, *,),
(i) C (Y) ⊆ T (Y, K(Y) ⊆ L(Y).
There exists k ∈ (0, 1) such that for every $\tilde{x}$, $\tilde{y}$ ∈ $\tilde{Y}$ and t > 0,
P (C $\tilde{x}$, K$\tilde{y}$, kt) ≥ min {P (L$\tilde{x}$, T$\tilde{y}$, t), P (C$\tilde{x}$, L$\tilde{x}$, t), P (K$\tilde{y}$, T$\tilde{y}$, t), P (C$\tilde{x}$, T$\tilde{y}$, t), P (C$\tilde{x}$,K$\tilde{y}$, t), P (L$\tilde{x}$, K$\tilde{y}$, t)},
and
Q (C $\tilde{x}$, K$\tilde{y}$, kt) ≤ max {Q (L$\tilde{x}$, T$\tilde{y}$, t), Q (C$\tilde{x}$, L$\tilde{x}$, t), Q (K$\tilde{y}$, T$\tilde{y}$, t), Q (C$\tilde{x}$, T$\tilde{y}$, t),Q (C$\tilde{x}$, K$\tilde{y}$, t), Q (L$\tilde{x}$, K$\tilde{y}$, t)}.
K and T weakly compatible mappings.
Then C, K, L, and T all have a single common soft fixed point in Y if the pair of mappings (C, L) is compatible of type (K) and one of the mappings is continuous.

Theorem 3.1: If the following requirements are met and B, K, L, and T are self mapping on a full intuitionistic soft fuzzy metric space (Y, P, Q, *,),
(i) C (Y) ⊆ T (Y), K(Y) ⊆ L(Y).
There exists k ∈ (0, 1) such that for every $\tilde{y}$, $\tilde{z}$ ∈ Y and t > 0,
P ($C_{\tilde{y}}$, $K_{\tilde{z}}$, kt) ≥ min{ P($L_{\tilde{y}}$, $T_{\tilde{z}}$,t), P($C_{\tilde{y}}$, $L_{\tilde{z}}$,t), P($K_{\tilde{z}}$, $T_{\tilde{z}}$,t), ($\frac{P(C_{\tilde{y}}, T_{\tilde{z}}, t)}{1+P(C_{\tilde{y}}, T_{\tilde{z}}, t)}$),

P ($C_{\tilde{y}}$
, $K_{\tilde{z}}$
, kt), $\left(\frac{P(L\tilde{y}\ ,K\tilde{z}\ ,t)}{1+P(L\tilde{y}\ ,K\tilde{z}\ ,t)}\right)$ } and

Q ($C_{\tilde{y}}$, $K_{\tilde{z}}$, kt) ≤ max{Q (L_y , $T_{\tilde{z}}$,t), Q (G_y , $L_{\tilde{z}}$,t), Q ($K_{\tilde{z}}$, $T_{\tilde{z}}$,t),
$\left(\frac{Q(C\tilde{y}\ ,T\tilde{z}\ ,t)}{1+Q(C\tilde{y}\ ,T\tilde{z}\ ,t)}\right)$, Q (C , K , kt), $\left(\frac{Q(L\tilde{y}\ ,K\tilde{z}\ ,t)}{1+Q(L\tilde{y}\ ,K\tilde{z}\ ,t)}\right)$

$\tilde{y}$ $\tilde{z}$

K and T weakly compatible mappings.
Then C, K, L, and T all have a single common soft fixed point in $\tilde{Y}$ if the pair of mappings (C, L) is compatible of type (K) and one of the mappings is continuous

Proof: Suppose $\tilde{y}_0$ be any random point in $\tilde{Y}$,constract a $\{\tilde{z}_n\}$ in $\tilde{Y}$
as a means to $\tilde{z}_{2\omega-1} = T\tilde{y}_{2\omega-1}$

= $C\tilde{y}_{2\omega-2}$ & $\tilde{z}_{2\omega} = L\,\tilde{y}_{2\omega} = K\,\tilde{y}_{2\omega+1}$, ω = 1,2,3,4 this can be done by (1),
By using contractive condition ,we obtain
P ($\tilde{z}_{2\omega+1}$, $\tilde{z}_{2\omega+2}$, kt) = P (C $\tilde{y}_{2\omega}$, $K\tilde{y}_{2\omega+1}$, kt)
P ($\tilde{z}_{2\omega+1}$, $\tilde{z}_{2\omega+2}$, kt) ≥ min{P($L\tilde{y}_{2\omega}$,T $\tilde{y}_{2\omega+1}$, t), P($C\tilde{y}_{2\omega}$, $L\tilde{y}_{2\omega+1}$, t), P($K\tilde{y}_{2\omega+1}$, T $\tilde{y}_{2\omega+1}$, t), P($L\tilde{y}_{2\omega}$,
$K\tilde{y}$
$2\omega+1$
, t), $\frac{P(L\tilde{y}2\omega\ ,K\tilde{y}2\omega+1\ ,t)}{1+P(L\tilde{y}2\omega\ ,\tilde{K}y2\omega+1\ ,t)}$,
$\frac{P\ (C\tilde{y}2\omega,\tilde{T}y2\omega+1\ ,t)}{1+P\ (\tilde{C}y2\omega,\tilde{T}y2\omega+1\ ,t)}$ }
≥ min{P($\tilde{z}_{2\omega}$, $\tilde{z}_{2\omega+1}$, t), P($\tilde{z}_{2\omega+1}$, $\tilde{z}_{2\omega+1}$, t), P ($\tilde{z}_{2\omega+1}$, $z_{2\omega+1}$, t), P($\tilde{z}_{2\omega}$, $\tilde{z}_{2\omega+1}$, t),

$\frac{P(z2\omega+1\ ,\tilde{z}2\omega+1\ ,t)}{1+P(z2\omega+1\ ,\tilde{z}2\omega+1\ ,t)}$,
$\frac{P\ (\tilde{z}2\omega\ ,\tilde{z}2\omega,t)}{1+P\ (\tilde{z}2\omega,z2\omega\ ,t)}$ }

P ($\tilde{z}_{2\omega+1}$, $\tilde{z}_{2\omega+2}$, kt) ≥ min{P($\tilde{z}_{2\omega}$, $\tilde{z}_{2\omega+1}$, t), P ($\tilde{z}_{2\omega+1}$, $\tilde{z}_{2\omega+1}$, t), P ($\tilde{z}_{2\omega+1}$, $z_{2\omega+1}$, t), P($\tilde{z}_{2\omega}$, $\tilde{z}_{2\omega+1}$, t),
$\frac{P(z2\omega+1\ ,\tilde{z}2\omega+1\ ,t)}{1+P\ (\tilde{z}2\omega\ ,\tilde{z}2\omega,t)}$ } P(z2ω+1 ,$\tilde{z}$2ω+1 ,t)
P ($\tilde{z}$2ω,z2ω ,t)
P ($\tilde{z}_{2\omega+1}$, $\tilde{z}_{2\omega+2}$, kt) ≥ min{P($\tilde{z}_{2\omega}$, $\tilde{z}_{2\omega+1}$, t), 1, 1, P($\tilde{z}_{2\omega}$, $\tilde{z}_{2\omega+1}$, t),1,1 }

P ($\tilde{z}_{2\omega+1}$, $\tilde{z}_{2\omega+2}$, kt) ≥ P($\tilde{z}_{2\omega}$, $\tilde{z}_{2\omega+1}$, t)
Similarly, we have P($\tilde{z}_{2\omega}$, $\tilde{z}_{2\omega+1}$, t) ≥ P($\tilde{z}_{2\omega-1}$, $\tilde{z}_{2\omega}$, t) (1) So we get P($\tilde{z}_{2\omega+2}$, $\tilde{z}_{2\omega+1}$, t) ≥ P($\tilde{z}_{2\omega+1}$, $\tilde{z}_{2\omega}$, t)
Also ,we get
Q($\tilde{z}_{2\omega+1}$, $\tilde{z}_{2\omega+2}$, kt) = Q($C\tilde{z}_{2\omega}$, $K\tilde{z}_{2\omega+1}$,kt)
Q($\tilde{z}_{2\omega+1}$, $\tilde{z}_{2\omega+2}$,kt) ≤ max{Q($L\tilde{y}_{2\omega}$,T $\tilde{y}_{2\omega+1}$, t), Q ($C\tilde{y}_{2\omega}$, $L\tilde{y}_{2\omega+1}$, t), Q ($K\tilde{y}_{2\omega+1}$, T $\tilde{y}_{2\omega+1}$, t),

Q($L\tilde{y}_{2\omega}$
, $K\tilde{y}$
$2\omega+1$
, t), $\frac{Q(L\tilde{y}2\omega\ ,\tilde{K}y2\omega+1\ ,t)}{1+Q(L\tilde{y}2\omega\ ,\tilde{K}y2\omega+1\ ,t)}$,
$\frac{Q\ (C\tilde{y}2\omega,T\tilde{y}2\omega+1\ ,t)}{1+Q\ (C\tilde{y}2\omega,\tilde{T}y2\omega+1\ ,t)}$ }

$\leq \max \{Q(\tilde{z}_{2\omega}, \tilde{z}_{2\omega+1}, t), Q(\tilde{z}_{2\omega+1}, \tilde{z}_{2\omega+1}, t), Q(\tilde{z}_{2\omega+1}, z_{2\omega+1}, t), Q(\tilde{z}_{2\omega}, \tilde{z}_{2\omega+1}, t),$

$$\frac{Q(z2\omega+1, \tilde{z}2\omega+1, t)}{1+Q(\tilde{z}2\omega, z2\omega, t)} Q(z2\omega+1, \tilde{z}2\omega+1, t), \frac{Q(\tilde{z}2\omega, \tilde{z}2\omega, t)}{1+Q(\tilde{z}2\omega, z2\omega, t)} \}$$

$Q(\tilde{z}_{2\omega+1}, \tilde{z}_{2\omega+2}, kt) \leq \max \{Q(z_{2\omega}, \tilde{z}_{2\omega+1}, t), Q(\tilde{z}_{2\omega+1}, z_{2\omega+1}, t), Q(\tilde{z}_{2\omega+1}, z_{2\omega+1}, t),$
$Q(\tilde{z}_{2\omega}, z_{2\omega+1}, t), \frac{Q(z2\omega+1, \tilde{z}2\omega+1, t)}{1+Q(\tilde{z}2\omega, z2\omega, t)} Q(z2\omega+1, \tilde{z}2\omega+1, t), \frac{Q(\tilde{z}2\omega, \tilde{z}2\omega, t)}{1+Q(\tilde{z}2\omega, z2\omega, t)} \}$

$Q(\tilde{z}_{2\omega+1}, z_{2\omega+2}, kt) \leq \{Q(z_{2\omega}, \tilde{z}_{2\omega+1}, t), 0, 0, Q(z_{2\omega}, z_{2\omega+1}, t), 0, 0\}$ $Q(\tilde{z}_{2\omega+1}, z_{2\omega+2}, kt) \leq Q(z_{2\omega}, \tilde{z}_{2\omega+1}, t)$

Similarly we have

$Q(\tilde{z}_{2\omega}, \tilde{z}_{2\omega+1}, t) \leq Q(\tilde{z}_{2\omega-1}, \tilde{z}_{2\omega}, t)$ (2) So we get $Q(\tilde{z}_{2\omega+2}, \tilde{z}_{2\omega+1}, t) \leq Q(\tilde{z}_{2\omega+1}, \tilde{z}_{2\omega}, t)$

From (1),(2) and (2.13), We comprehend that $\{z\}$ is a Cauchy sequence in Y .

But $(\tilde{Y}, \tilde{P}, \tilde{Q}, *, \diamond)$ is complete,So there are a soft point $\tilde{z}$ in $\tilde{Y}$ .Such that $\{\tilde{z}_{\omega}\} \rightarrow \tilde{q}$. Also we have $\{C\tilde{y}_{2\omega-2}\}, \{T\tilde{y}_{2\omega-1}\}, \{L\tilde{y}_{2\omega}\}, \{K\tilde{y}_{2\omega-1}\} \rightarrow \tilde{q}$.

Since (C, L) is compatible of type (K) and one of the mapping is continuous. So using (2.15),We obtain $C\tilde{q} = L\tilde{q}$. (3)

Since $C\tilde{(Y)} \subseteq T\tilde{(Y)}$,Then one can find a point $\tilde{v}$. in $\tilde{Y}$, as a means to $C\tilde{q} = T\tilde{v}$.

Now by contractive condition, we obtain

$$P(C\tilde{q}, K\tilde{v}, kt) \geq \min \{P(L\tilde{q}, T\tilde{v}, t), P(C\tilde{q}, L\tilde{v}, t), P(K\tilde{v}, T\tilde{v}, t), \frac{P(L\tilde{q}, K\tilde{v}, t)}{1+(L\tilde{q}, v, t)}, P(C\tilde{q}, K\tilde{v}, t), \frac{P(C\tilde{q}, T\tilde{v}, t)}{1+(C\tilde{q}, \tilde{v}, t} \}$$

$$= \min \{P(C\tilde{q}, C\tilde{q}, t), P(C\tilde{q}, C\tilde{q}, t), P(K\tilde{v}, C\tilde{q}, t), \frac{P(C\tilde{q}, K\tilde{v}, t)}{1+(C\tilde{q}, v, t)}, P(C\tilde{q}, C\tilde{q}, t), \frac{P(C\tilde{q}, K\tilde{v}, t)}{1+(C\tilde{q}, \tilde{v}, t} \}$$

$$= \min \{P(C\tilde{q}, C\tilde{q}, t), P(C\tilde{q}, C\tilde{q}, t), P(K\tilde{v}, C\tilde{q}, t), P(K\tilde{v}, C\tilde{q}, t), P(C\tilde{q}, C\tilde{q}, t), P(K\tilde{v}, C\tilde{q}, t)\}$$

$P(C\tilde{q}, K\tilde{v}, kt) \geq P(C\tilde{q}, K\tilde{v}, t)$ (4)

Also ,we obtain

$$Q(C\tilde{q}, K\tilde{v}, kt) \leq \max \{Q(L\tilde{q}, T\tilde{v}, t), Q(C\tilde{q}, L\tilde{v}, t), Q(K\tilde{v}, T\tilde{v}, t), \frac{Q(L\tilde{q}, K\tilde{v}, t)}{1+Q(L\tilde{q}, \tilde{K}v, t)}, Q(C\tilde{q}, K\tilde{v}, t), \frac{Q(C\tilde{q}, T\tilde{v}, t)}{1+Q(C\tilde{q}, T\tilde{v}, t} \}$$

$$= \max \{Q(C\tilde{q}, C\tilde{q}, t), Q(C\tilde{q}, C\tilde{q}, t), Q(K\tilde{v}, C\tilde{q}, t), \frac{Q(C\tilde{q}, K\tilde{v}, t)}{1+Q(C\tilde{q}, K\tilde{v}, t)}, Q(C\tilde{q}, C\tilde{q}, t), \frac{Q(C\tilde{q}, \tilde{K}v, t)}{1+Q(C\tilde{q}, K\tilde{v}, t} \}$$

= max { Q(Cq̃,Cq̃,t),Q(Cq̃,Cq̃,t),Q(Kṽ,Cq̃,t), Q(Kṽ,Cq̃,t) ,Q(Cq̃,Cq̃,t), Q(Kṽ,Cq̃,t) }

Q(Cq̃,Kṽ,kt) ≤ Q(Cq̃,Kṽ,t) (5)

Now from (4) and (5) and Lemma (2.14), We obtain Cq̃ = Kṽ
Thus we obtain Cq̃ = Lq̃ =Kq̃ =Tq̃ (6)
To prove Rq̃ = q̃
P(Cq̃, $Kz_{2\omega+1}$ kt) ≥min {P(Lq̃, $Tz_{2\omega+1}$, t) , P(Cq̃, $Lz_{2\omega+1}$, t) ,P($Kz_{2\omega+1}$, $Tz_{2\omega+1}$, t),

$\frac{P(Lq̃,Tz_{2\omega+1},t)}{1+P(Lq,Tz_{2\omega+1},t)}$
, P(Cq̃, Kz
$_{2\omega+1}$
, t) , $\frac{P(Cq̃,Tz_{2\omega+1},t)}{1+P(Cq̃,Tz_{2\omega+1},t)}$ }

let as n → ∞, we obtain

P(Cq̃, q̃,kt) ≥ min {P(Lq̃, q̃,t),P (Cq, q̃,t),P(q̃, q̃,t), $\frac{P(Lq̃,q̃,t)}{1+P(Lq̃,q̃,t)}$

, P(Cq̃, q̃,t), $\frac{P(Cq̃,q̃,t)}{1+P(Cq̃,q̃,t)}$ }
P(Cq̃, q̃,kt) = min {P(Lq̃, q̃,t), P (Cq̃, q̃,t), 1 , $\frac{P(Cq̃,q̃,t)}{1+P(Cq̃,q̃,t)}$
, P(Cq̃, q̃,t), $\frac{P(Cq̃,q̃,t)}{1+P(Cq̃,q̃,t)}$ }

P(Cq̃, q̃,kt) = min {P(Cq̃, q̃,t),P (Cq̃, q̃,t),1 , $\frac{P(Cq̃,q̃,t)}{1+P(Cq̃,q̃,t)}$
, P(Cq̃, q̃,t), $\frac{P(Cq̃,q̃,t)}{1+P(Cq̃,q̃,t)}$ }

P(Cq̃, q̃,kt) = min {P(Cq̃, q̃,t),P (Cq̃, q̃,t),1 ,1 , P(Cq̃, q̃,t), 1 }

P(Cq̃, q̃,kt) ≥ P (Cq̃, q̃,t) (7)

Q(Cq̃, $Kz_{2\omega+1}$ kt) ≤ max {Q(Lq̃, $Tz_{2\omega+1}$, t) , Q(Cq̃, $Lz_{2\omega+1}$, t) ,Q($Kz_{2\omega+1}$, $Tz_{2\omega+1}$, t),

$\frac{Q(Lq̃,Tz_{2\omega+1},t)}{1+Q(Lq̃,Tz_{2\omega+1},t)}$,Q(Cq̃, Kz

let as n → ∞, we obtain
$_{2\omega+1}$
, t) , $\frac{Q(Cq̃,Tz_{2\omega+1},t)}{1+Q(Cq̃,Tz_{2\omega+1},t)}$ }

Q(Cq̃, q̃,kt) ≤max {Q(Lq̃, q̃,t), Q(Cq̃, q̃,t), Q(q̃, q̃,t), $\frac{Q(Lq̃,q̃,t)}{1+Q(Lq̃,q̃,t)}$
, Q(Cq̃, q̃,t), $\frac{Q(Cq̃,q̃,t)}{1+Q(Cq̃,q̃,t)}$ }

$Q(C\tilde{q}, \tilde{q},kt) = \max \{Q(L\tilde{q}, \tilde{q},t), Q(C\tilde{q}, \tilde{q},t), 0, \frac{Q(C\tilde{q},\tilde{q},t)}{1+Q(C\tilde{q},\tilde{q},t)}, Q(C\tilde{q}, \tilde{q},t), \frac{Q(C\tilde{q},\tilde{q},t)}{1+Q(C\tilde{q},\tilde{q},t)}\}$

$Q(C\tilde{q}, \tilde{q}, kt) = \max\{Q(C\tilde{q}, \tilde{q},t), Q(C\tilde{q}, \tilde{q},t), 0, \frac{Q(C\tilde{q},\tilde{q},t)}{1+Q(C\tilde{q},\tilde{q},t)}, Q(C\tilde{q}, \tilde{q},t), \frac{Q(C\tilde{q},\tilde{q},t)}{1+Q(C\tilde{q},\tilde{q},t)}\}$

$Q(C\tilde{q}, \tilde{q}, kt) = \max \{Q(C\tilde{q}, \tilde{q},t), Q(C\tilde{q}, \tilde{q},t), 0, 0, Q(C\tilde{q}, \tilde{q},t), 0\}$

$Q(C\tilde{q}, \tilde{q},kt) \leq \max Q(C\tilde{q}, \tilde{q},t)$ (8)

Now from (7) and (8) & lemma 2.14, we obtain

$C\tilde{q} = \tilde{q}$

That being the case ,we obtain $C\tilde{q} = L\tilde{q} = \tilde{q}$ So $\tilde{q}$ is a comman soft point of C and L. (9)

As well we obtain $K\tilde{v} = T\tilde{v} = \tilde{q}$ (10)

Since K and T are weakly compatible ,we have $T K\tilde{v} = K T\tilde{v}$.

So from (6),we obtain $T\tilde{q} = K\tilde{q}$ Again ,we obtain (11)

$P(C\tilde{z}_{2\omega-2}, K\tilde{q}, kt) \geq \min\{P(L\tilde{z}_{2\omega-2}, T\tilde{q},t), P(C\tilde{z}_{2\omega-2}, L\tilde{q}, t), P(K\tilde{q},T\tilde{q},t), \frac{P(C\tilde{z}_{2\omega-2}, T\tilde{q},t)}{1+P(C\tilde{z}_{2\omega-2}, T\tilde{q},t)},$

$P(C\tilde{z}_{2\omega-2}, L\tilde{q}, t), \frac{P(C\tilde{z}_{2\omega-2}, T\tilde{q},t)}{1+P(C\tilde{z}_{2\omega-2}, T\tilde{q},t)}\}$

Taking limit $n \to \infty$,We have

$P(\tilde{q}, K\tilde{q}, kt) \geq \min\{P(\tilde{q},T\tilde{q},t), P(\tilde{q}, L\tilde{q}, t), P(K\tilde{q},T\tilde{q},t), \frac{P(\tilde{q}, T\tilde{q},t)}{1+P(\tilde{q}, T\tilde{q},t)}\}$

$P(\tilde{q}, K\tilde{q}, kt) \geq \min\{P(\tilde{q},T\tilde{q},t), P(\tilde{q}, T\tilde{q}, t), P(T\tilde{q},T\tilde{q},t), \frac{P(\tilde{q}, T\tilde{q},t)}{1+P(\tilde{q}, T\tilde{q},t)}, P(\tilde{q}, T\tilde{q}, t), \frac{P(\tilde{q}, T\tilde{q},t)}{1+P(\tilde{q}, T\tilde{q},t)}, P(T\tilde{q}, T\tilde{q}, t), \frac{P(\tilde{q}, T\tilde{q},t)}{1+P(\tilde{q}, T\tilde{q},t)}\}$

$P(\tilde{q}, K\tilde{q}, kt) \geq \min\{P(\tilde{q},T\tilde{q},t), P(\tilde{q}, T\tilde{q}, t), 1, P(\tilde{q}, T\tilde{q}, t), 1, P(\tilde{q}, T\tilde{q}, t)\}$

$P(\tilde{q}, K\tilde{q}, kt) \geq P(\tilde{q},T\tilde{q},t)$ (12)

As well ,we obtain

$Q(C\tilde{z}$ $Q(C\tilde{z}$

2ω−2

2ω−2
, Kq̃, kt)≤ max {Q(Lž
, Lq̃, t), Q(Cž2ω−2 ,Tq̃,t) }
1+ Q(Cž2ω−2 ,Tq̃,t)
2ω−2
,Tq̃,t), Q(Cž
2ω−2
, Lq̃, t),Q(Kq̃,Tq̃,t), Q(Cž2ω−2 ,Tq̃,t) ,
1+ Q(Cž2ω−2 ,Tq̃,t)

Taking limit n → ∞ ,We have
Q(q̃ , Kq̃, kt) ≤ max{Q(q̃,Tq̃,t), P(q̃ , Lq̃, t),Q(Kq̃,Tq̃,t), Q(q̃ ,Tq̃,t)
1+ Q(q̃ ,Tq̃,t)

Q(q̃ , Kq̃, kt) ≤ max {Q(q̃,Tq̃,t), Q(q̃ , Tq̃, t),Q(Tq̃,Tq̃,t), Q(q̃ ,Tq̃,t)
1+ Q(q̃ ,Tq̃,t)
, Q(q̃ , Tq̃, t), Q(q̃ ,Tq̃,t) }
1+ Q(q̃ ,Tq̃,t)

, Q(Tqq̃, Tq̃, t), Q(q̃ ,Tq̃,t) }
1+ Q(q̃ ,Tq̃,t)

Q(q̃ , Kq̃, kt) ≤ max {Q(q̃,Tq̃,t), Q(q̃ , Tq̃, t), 1 , Q(q̃, Tq̃ , t), 1, Q(q̃ , Tq̃ , t)}

Q(q̃ , Kq̃ , kt) ≤ Q(q̃,Tq̃,t) (13)
Now from (12) and (13) and Lemma(2.14)
Since we have Tq̃ = Kq̃ = q̃ (14)
That being the case ,we obtain q̃ is a ordinary soft fixed point of C,K,L and T. For Uniqueness
Suppose r̃ be the another ordinary soft fixed point then Cr̃ = Lr̃ = Kr̃ Rr̃ = r̃

P(Cq̃, K̃r, kt) ≥ min {P((Lq̃, Tr̃, t), P(Cq̃, Lr̃, t) , P(K̃r, Tr̃, t), P(Cq̃,Tr̃,t)
1+ P(Cq̃,Tr̃,t)
, P(Cq̃, Kr̃, t) , P(Lq̃,Tr̃,t) }
1+P (Lq̃,Tr̃,t)

P(Cq̃, K̃r, kt) ≥ min {P((Cq̃, Kr̃, t), P(Cq̃, C̃r, t) , P(K̃r, Kr̃, t), P(Cq̃,Kr̃,t)
1+P (Cq̃,Kr̃,t)
, P(Cq̃, Kr̃, t) , P(Cq̃,Kr̃,t) }
1+P (Lq̃,Kr̃,t)
P(Cq̃, Kr̃, kt) = min {P((Cq̃, K̃r, t), P(Cq̃, Cr̃, t) , 1, , P(Cq̃, Kr̃, t) , P(Cq̃, K̃r, t), P(Cq̃, Kr̃, t) }

(Cq̃, Kr̃, kt) ≥ P((Cq̃, K̃r, t) (15)
As well we obtain

Q(Cq̃, Kr̃, kt) ≤ max {Q((Lq̃, T̃r, t), Q(Cq̃, Lr̃, t) , Q(Kr̃, T
, t), Q(Cq̃,Tr̃,t)
1+ Q(Cq̃,Tr̃,t)
, Q(Cq̃, Kr̃, t) , Q(Lq̃,Tr̃,t) }
1+Q (Lq̃,Tr̃,t)

Q(Cq̃, Kr̃, kt) ≤ max {Q((Cq̃, Kr̃, t), Q(Cq̃, C̃r, t) , Q(Kr̃, K̃r, t), Q(Cq̃,Kr̃,t)
1+Q (Cq̃,Kr̃,t)
}

, Q(Cq̃, Kr̃, t) , $\frac{Q(Cq̃,Kr̃,t)}{1+ Q(Lq̃,Kr̃,t)}$

Q(Cq̃, Kr̃, kt) ≤ max{Q((Cq̃, K
, t), Q(Cq̃, Cr̃, t) , 0, , Q(Cq̃, Kr̃, t) , Q(Cq̃, K
, t), Q(Cq̃, Kr̃, t) }

Q(Cq̃, K̃r, kt) ≤ Q((Cq̃, Kr̃, t) (16)
From (15),(16) and Lemma(2.14)
We obtain Cq̃ = Kq̃ ,This implies Cq̃ = Cr̃ ,and q̃ is a unique soft fixed point.

Remarks: Our result is also true for the pair of mappings (A, S) is compatible of type (E) in place of compatible mapping of type (K). Our result extends and generalizes the results of M. Verma and R.S. Chandel [13], Jha *et al.*[11] Manandhar *et al.* and improve the result of Manandhar *et al.* [10]. Also, our result improves other similar results in literature.

References

1. George, P. Veeramani, On some results in fuzzy metric spaces Fuzzy Sets and Systems, **64**
2. (1994), 395-399.
3. Schweizer, Sklar, "A. Statistical metric spaces", Pacific J. of Math., 10 (1960) 314-334.
4. Alaca, Turkoglu.D, Yildiz.C, "Fixed points in intuitionistic fuzzy metric Spaces", Chaos, Solitons and Fractals, **29**(2006), 1073-1078.
5. G. Jungck and B. E. Rhoades, Fixed Point for Set Valued functions without Continuity, Indian J. Pure Appl. Math., 29(3), (1998), 771- 779.
6. G. Jungck, Compatible mappings and common fixed points, Internat. J. Math. Math. Sci.,
7. **9**(1986), pp. 771–779.
8. G. Jungck, P. P. Murthy and Y. J. Cho, Compatible mappings of type (A) and common fixed points, Math. Japonica, **38**(1993), 381–390.
9. H. K. Pathak, Y. J. Cho, S. S. Chang and S. M. Kang, Compatible mappings of type (P) and fixed point theorem in metric spaces and Probabilistic metric spaces, Novi Sad J. Math., Vol. 26(2)(1996), 87-109.
10. J.H. Park, "Intuitionistic fuzzy metric spaces", Chaos, Solitons & Fractals **2**(2004), 1039–1046. [9]. K. B. Manandhar, K. Jha and G. Porru. Common Fixed Point Theorem of Compatible
11. Mappings of Type (K) in Fuzzy Metric Space, Electronic J. Math. Analysis and Appl,
12. **2**(2)(2014), 248-253.
13. [10]. K. B. Manandhar, K. Jha and H. K. Pathak, A Common Fixed Point Theorem for mpatible
    Mappings of Type (E) in Fuzzy Metric space, Appl Math. Sci, **8** (2014), 2007 – 2014.
14. K. Jha, V. Popa and K.B. Manandhar, A common fixed point theorem for compatible mapping of type (K) in metric space, Internat. J. of Math. Sci. & Engg. Appl. (IJMSEA), **8** (I) (2014), 383-391.
15. L.A. Zadeh, Fuzzy sets, Inform and Control **8** (1965), 338-353.
16. M. Verma and R. S. Chandel, Common fixed point theorem for four mappings in intuitionistic fuzzy metric space using absorbing Maps, IJRRAS 10 (2) (2012), 286 – 291.
17. [14]. O. Kramosil and J. Michalek, Fuzzy metric and statistical metric spaces, Kybernetika **11**
18. (1975), 326-334.
19. R. P. Pant, A common fixed point theorem under a new condition, Indian. J. Pure Appl. Math., 30 (2) (1999) 147–152.
20. S. Sharma., Kutukcu,and R.S. Rathore , "Common fixed point for Multivalued mappings in intuitionistic fuzzy metricspace", Communication of Korean Mathematical Society, **22** (3),(2007), 391-399.
21. Y.J. Cho, H.K. Pathak, S.M. Kang, J.S. Jung, Common fixed points of compatible maps of type (β) on fuzzy metric spaces, Fuzzy Sets and Systems **93** (1998), 99-111.
22. Molodtsov, D. **Soft –theory-first result**, Comput.Math.Appl. 37,19-31 (1999).
23. Moji, P.K. Roy, A.R, Bisbas,R.**An application of soft sets in a decision making problem**, Comput. Math. Appl. 44, 1077 – 1083 (2002).
24. Moji, P.K.Roy, A.R,Bisbas,R.**Soft set theory**, Computer. Math. Appl. 45, 555 – 562(2003).
25. Ali, M.I., Feng, F., Liu, X., Min, W.K., Shabir, M.: **On some new operations in soft settheory**. Computer. Math. Appl. 57, 1547–1553 (2009).
26. C¸aˇgman, N., Enginoglu, S.: **Soft set theory uni-int decision making**. Eur. J. Oper. Res. 207, 845–855 (2010).
27. Babitha, K.V., Sunil, J.J.: **Soft set relations and functions**. Comput. Math., Appl. 60, 1840–1849 (2010).

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Abstract

Objective/Aim: To demonstrate the existence and uniqueness of fixed points for self-maps in b-metric spaces, we utilized a generalized ϕ-weak contractive condition that incorporates cubic terms of d(x, y), along with the weak compatibility of two mappings within the context of b-metric spaces. Our findings include the proof of various fixed point theorems for a self-map, as well as common, point theorems for two mappings, accompanied by appropriate examples to substantiate the established results. The innovative aspect of our work lies in proving the existence of fixed points for mappings that meet generalized ϕ-weak contractive conditions with cubic terms of d(x, y) in b-metric spaces, a result that has not been previously established by others. Furthermore, our results serve to extend,discribe and generalize the findings of Dutta and Choudhury, Murthy and Prasad, Hao and Guan, Jain and Kaur, Petru, sel, A. and Petru, sel, J., Peng et al., among others.Ues for other fixed point theorem and improve with work.

Mathematics Subject Classification: *54H25,47H10.*

Keywords*: Fixed Point, b-Metric Space, α–Weak Contraction, Generalized μ–Weak Contraction, Weakly Compatible Mapping, Soft Set, Soft Fixed Point.*

Introduction

Fixed point Theory is one of the most important and useful tool in many branches of Science, Economics, Engineering and Computer Science, etc. The majority of applied mathematics problems reduce to fixed point problems. The famous Banach contraction principle proves the existence and uniqueness of fixed points for a contraction mapping in complete metric space. It has been generalized by many researchers by finding fixed point of single map or common fixed points of two or more mapping in various spaces satisfying various contractive conditions which are either commuting or weakly commuting or compatible or weakly compatible etc. In the development of various metric spaces, one of the significant generalizations of metric space is b- metric space introduced by Bakhtin[(1)]in 1989. Afterwards, Czerwik[(2)]proved some fixed point theorems in b-metric spaces in1993.

Theorem1.1: " Let (Z, d) be a complete metric space and S be a φ – contraction mapping on a Z . i. e $d(Sp ,Sq) \leq \varphi \{ d(p, q)\}$ for all p, q $\in$ Z where φ: $[0,\infty) \rightarrow [0, \infty)$ is an upper semi-continuous function from right such that $0 \leq \varphi(t) < t$ for all $t > 0$. Then S has a unique fixed point."

In 1997 Alber and Guerre-Delabriere introduced φ – weak contraction and proved the following.

Theorem1.2: Let (Z, d) be a complete metric space and φ – weak contraction mapping on Z. i.e. , $d(Sp ,Sq) \leq d(p, q) - \varphi \{ d(p, q)\}$ for all p, q $\in$ Z, where φ: $[0,\infty) \rightarrow [0, \infty)$ is an upper semi-continuous function from right such that $\varphi(t) > 0$ for all $t > 0$and $\varphi(0) = 0$. Then S has a unique fixed point.

Theorem 1.3 : Let (Z, d) be a complete metric and S be a self map of Z satisfying the following condition:

$[1 + rd(p, q)] \; d^2(Sp, Sq) \leq r \max \{ \frac{1}{2} [d^2(p,Sp)\, d(q,Sq) + d(p,Sp) d^2(q,Sq)], [d(p,Sp) d(p,Sq) d(q,Sp)], \; d(p, Sq)\, d(q, Sp)\, d(q, Sq)\}$

Where

k(p,q) =

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$$\max\left\{d^2(p,q), d(p,Sp)d(q,Sp),\ d(q,Sp)d(p,Sp),\ \frac{1}{2}[d(p,Sp)d(p,Sq) + d(q,Sp)d(q,Sq),]\right\}$$

In the present paper, we generalize the above result in the setting of b-metric space and extend our result for a pair of weakly compatible self mappings in b-metric space. We also obtain some suitable examples to justify the proven results. Our results extend and generalize the results of Dutta and Choudhury(13), Murthy and Prasad(12), Hao and Guan(7), Jain and Kaur(8), Petrusel, A. and Petrusel, J.(10), Peng et. al.(11)etc.

Preliminaries:

We now give some known definitions and standard notations which will be needed in the sequel:

Definition2.1

Let a nonempty set Z and let s≥1 be a given real number. A function d:Z×Z→⌊0,∞) is said to be b-metric if the following conditions hold good:

(1) d(g ,h) = 0 if and only if g = h.

(2) d(g ,h) = d(h, g).

(3) d(g ,h) ≤ s⌊d(g ,r)+d(r, h)⌋, for all g, h and r ∈ Z.

Then (Z ,d ,s) is called a **b-metric** or metric type space.

Definition 2.2

"The b-metric space (Z, d, s) is called **complete** if every Cauchy sequence in z is convergent in Z."

Definition 2.3

"Let (Z, d, s) be a b-metric space and S,T:Z→Z be two mappings. The mapping S and T are said to be **weakly compatible** if they commute at their coincident points, i.e., TSp = STp whenever Sp = Tp."

Main Results:

Theorem 3.1.

Let (Z, d, s) be a complete b-metric space and T be a self-map of Z satisfying

$$\lceil 1 + rd(p,q)\rceil\ d^2(Sp,Sq) \le \frac{r}{s}\max\ \{\frac{1}{2}\ [d^2(p,Sp)\ d(q,Sq) + d(p,Sp)\ d^2(q,Sq)\ ,\frac{d^2(q,Sp)}{1+d(q,Sp)}\rceil,\lceil d(p,Sp)\ d(p,Sq)\ d(q,Sp) + \frac{d^2(p,Sq)}{1+d(p,Sq)(q,Sp)}\rceil, d(p,Sq)\ d(q,Sp)\ d(q,Sq)\ \} + m(p,q) - \varphi\{m(p,q)\} \qquad (3.1.1)$$

Where

$$k(p,q) = \max\left\{d^2(p,q), d(p,Sp)d(q,Sp)\frac{d^2(q,Sp)}{1+d(q,Sp)}\ ,\ d(q,Sp)d(p,Sp),\ \frac{1}{2s}[d(p,Sp)d(p,Sq),\left(\frac{d^2(q,Sp)}{1+d(q,Sp)}\right) + d(q,Sp)d(q,Sq),\left(\frac{d^2(p,Sq)}{1+d(p,Sp)(q,Sp)}\right)]\right\}$$

r ≥ 0 is a real number and φ:⌊0,∞)⟶ ⌊0,∞) is a continuous function with φ (t) = 0 if and only if t = 0 and φ (t) > 0 for each t > 0. Then, T has a unique fixed point in Z.

Proof: Let p_0 be an arbitrary point in Z.We define a sequence $\{p_n\}$in Z .$p_{n+1} = T\,p_n$, n ≥ 0. (3.1.2)

If $p_n = p_{n+1}$, for some n, then, T has trivially a fixed point. Therefore we assume that $p_n \neq p_{n+1}$ for all n∈ N .For convenience

$d(p_n,p_{n+1}) = d_n$ (3.1.3)

First,we show that $\{d_n\}$ is a monotonically decreasing sequence, i.e., $d_{n+1} \le d_n$.

Case1: If n is odd. Take n = 2m-1. We prove that $d_2 \le d_{2m-1}$.

Now taking $p = p_{2m-1}, q = p_{2m}$ in equality (3.1.1)

we have $\lceil 1+ r\ d(p_{2m-1}, p_{2m})\rceil\ d^2(p_{2m-1},Sp_{2m}) \le \frac{r}{s}$ max $\frac{1}{2}$ $[d^2(p_{2m-1}, S\,p_{2m-1})\ d(p_{2m}, Sp_{2m}) + d(p_{2m-1},Sp_{2m-1})d^2(p_{2m},Sp_{2m}),\frac{d^2(p_{2m},Sp_{2m-1}}{1+d(p_{2m},Sp_{2m-1})}\rceil,\lceil d(p_{2m-1},Sp_{2m-1})d(p_{2m-1},Sp_{2m})\ d(p_{2m},Sp_{2m-1}) + \frac{d^2(p_{2m},Sp_{2m-1})}{1+d(p_{2m},Sp_{2m-1})d(p_{2m-1},S\,p_{2m}}\rceil,d(p_{2m-1},Sp_{2m})\ d(p_{2m},Sp_{2m})\ d(p_{2m},Sp_{2m})\ \} + m(p_{2m-1}, p_{2m})$ -

$\varphi\ \lceil m(p_{2m-1}, p_{2m})\rceil$

Where

$$K(p_{2m-1}, p_{2m}) = \max\left\{ \begin{array}{c} d^2(p_{2m-1}, p_{2m}), d(p_{2m-1}, Sp_{2m-1})d(p_{2m}, Sp_{2m-1}) \\ \left(\frac{d^2(p_{2m},\ Sp_{2m-1})}{1+ d(p_{2m}, Sp_{2m-1})}\right), d(p_{2m}, Sp_{2m-1})d(p_{2m-1}, Sp_{2m-1}), \\ \frac{r}{2s}\left[d(p_{2m-1}, Sp_{2m-1})d(p_{2m-1}, Sp_{2m})\left(\frac{d^2(p_{2m}, Sp_{2m})}{1+ d(p_{2m}, Sp_{2m})}\right) + \right. \\ \left. d(p_{2m}, Sp_{2m-1})d(p_{2m}, Sp_{2m}), \left(\frac{d^2(p_{2m-1}, Sp_{2m})}{1+ d(p_{2m-1}, Sp_{2m-1})d(p_{2m-1}, Sp_{2m})}\right)\right] \end{array} \right\}$$

Now using Inequality (3.1.2)

$[1+ r\ d(p_{2m-1}, p_{2m})]\ d^2\ (p_{2m}, p_{2m+1}) \le \frac{r}{s}$ max $\{ \frac{1}{2}\ [d^2\ (p_{2m-1}, p_{2m})\ d(p_{2m}, p_{2m+1}) + d(p_{2m-1}, p_{2m})\ d^2\ (p_{2m}, p_{2m+1})$ $, \frac{d^2(p_{2m}, p_{2m})}{1+ d(p_{2m}, p_{2m})}\ ],\ [\ d(p_{2m-1}, p_{2m})\ d(p_{2m-1}, p_{2m+1})\ d(p_{2m}, p_{2m}) + \frac{d^2(p_{2m}, p_{2m})}{1+ d(p_{2m}, p_{2m})d(p_{2m-1}, p_{2m}}],d(p_{2m-1}, p_{2m+1})\ (p_{2m}, p_{2m})$ $d(p_{2m}, p_{2m+1})\ \} + m(p_{2m-1}, p_{2m}) - \varphi\ [m(p_{2m-1}, p_{2m})]$

$$k(p_{2m-1}, p_{2m}) = \max\left\{ \begin{array}{c} d^2(p_{2m+1}, p_{2m}), d(p_{2m-1}, p_{2m})d(p_{2m}, p_{2m})\left(\frac{d^2(p_{2m},\ p_{2m})}{1+ d(p_{2m},\ p_{2m})}\right) \\ , d(p_{2m}, p_{2m})\ d(p_{2m-1}, p_{2m}), \\ \frac{r}{2s}\left[d(p_{2m-1}, p_{2m})\ d(p_{2m-1}, p2m+1), \left(\frac{d^2(p_{2m}, p_{2m+1})}{1+ d(p_{2m}, p_{2m})}\right) + d(p_{2m}, p_{2m+1}), \right. \\ \left. \left(\frac{d^2(p_{2m-1}, p_{2m+1})}{1+ d(p_{2m-1}, p2m)\ d(p_{2m-1}, p_{2m+1})}\right)\right] \end{array} \right\}$$

$[1+ r\ d_{2m-1}\ ]\ d^2_{2m} \le \frac{r}{s}$ max $\{ \frac{1}{2}\ [d^2_{2m-1}\ d_{2m} + d_{2m-1}d^2_{2m}\ ,\ 0,\ 0,\ 0\} + m(d_{2m-1}, d_{2m}) - \varphi\ (d_{2m-1}, d_{2m})]$ (3.1.4)

Where

$k(p_{2m-1}, p_{2m},) = \max\left\{d^2_{2m-1}, d^2_{2m-1}\ d_{2m}\ d_{2m}\ 0,\ \frac{r}{2s}\ [d^2_{2m-1}\ d(p_{2m-1}, p_{2m+1}), 0\ ,0]\right\}$

$k(p_{2m-1}, p_{2m}) = \max\left\{d^2_{2m-1}, d^2_{2m-1}\ d_{2m}\ d_{2m}\ 0,\ \frac{r}{2s}\ [d^2_{2m-1}\ r(d^2_{2m-1} + d^2_{2m}\ ),0\ ,0]\right\}$

[By triangular inequality of b-metric space]

If $d^2\ 2m - 1 < d2m$

Then

$m(p2m - 1, p2m) < \max\ \{ d^2_{2m}\ ,0,0,\ d^2_{2m}\} = d^2_{2m}$

So Inequality (3.1.4) becomes

$d^2_{2m} + r\ d^2_{2m-1}\ d^2_{2m} < \frac{r}{s} \max\{ \frac{1}{2}\ [d^2_{2m-1}\ d^2_{2m} + d^2_{2m-1}\ d^2_{2m}\ ]\ ,0,0\} + d^2_{2m} - \varphi(d^2_{2m})$.

$\le r\ d^2_{2m-1}\ d^2_{2m} + d^2_{2m} - (d^2_{2m})$.

Which gives $\varphi(d^2_{2m}) < 0$, a contraction .

Thus $d^2_{2m} \le d^2_{2m-1}$

Case 2 : If n is even, taking n = 2m and p = p_{2m}

q = p_{2m+1} in Inequality (1) and proceeding as above we can easily obtain that $d^2_{2m+1} \le d^2_{2m}$

Therefore ,$\{d_n\}$ is a monotonically decreasing sequence of non-negative real numbers and convergent.

Let $\log_{n\to\infty} d_n = k$ for some $k \ge 0$ (3.1.5)

We assume that k = 0. Using Inequality (3.1.1), we have

$[1+ r\ d(p_n, p_{n+1})]\ d^2\ (Sp_n, Sp_{n+1}) \le \frac{r}{s}$ max $\frac{1}{2}[d^2\ (p_n, Sp_n)\ d(p_{n+1}, S\ p_{n+1}) + d(p_n, Sp_n)$

$d^2(p_{n+1}, Sp_{n+1}), \frac{d^2(p_{n+1}, Sp_n}{1+ d(p_{n+1}, Sp_n)}],[d(p_n, Sp_n)\ d(p_n, S\ p_{n+1})d(p_{n+1}, Sp_n) + \frac{d^2(p_{n+1}, Sp_n)}{1+ d(p_{n+1}, Sp_n)d(p_n, S\ p_{n+1})}\ ],\ d(p_n, Sp_{n+1})\ d(p_{n+1}, Sp_n)$ $d(p_{n+1}, S\ p_{n+1})\ \} + m(p_n, p_{n+1}) - \varphi\ [m(p_n, p_{n+1})]$

$[1+ r\ d(p_n, p_{n+1})]\ d^2\ (p_{n+1}, p_{n+2}) \le \frac{r}{s}$ max $\frac{1}{2}\ [d^2\ (p_n, p_{n+1})\ d(p_{n+1}, p_{n+2}) + d(p_n, p_{n+1})\ d^2\ (p_{n+1}, p_{n+2})$ $, \frac{d^2(p_{n+1}, p_{n+1}}{1+ d(p_{n+1}, p_{n+1}}], [d(p_n, p_{n+1})\ d(p_n, p_{n+2})\ d(p_{n+1}, p_{n+1}) + \frac{d^2(p_{n+1}, p_{n+1})}{1+ d(p_{n+1}, p_{n+1})d(p_n, p_{n+2})}],$

$d(p_n, p_{n+2})\ d(p_{n+1}, p_{n+1})\ d(p_{n+1}, p_{n+2})\ \} + m(p_n, p_{n+1}) - \varphi\ [m(p_n, p_{n+1})]$

This implies that

$(1+ r\ d_n)\ d^2_{n+1} \le \frac{r}{s} \max\ \{ \frac{1}{2}\ [d^2_n\ d_{n+1} + d_n d^2_{n+1}, 0],0,0\ \} + m\ (p_n, p_{n+1}) - \varphi\ [m\ (p_n, p_{n+1})]$ (3.1.6)

Where

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$$k(p_n, p_{n+1}) = \max\left\{\begin{matrix} d^2(p_n, p_{n+1}),\ d(p_n, Sp_n)d(p_{n+1}, Sp_n) \\ \left(\frac{d^2(p_{n+1}, Sp_n)}{1+d(p_{n+1}, Sp_n)}\right), d(p_{n+1}, Sp_n)d(p_n, Sp_n), \\ \frac{r}{2s}\left[d(p_n, Sp_n)d(p_n, Sp_{n+1}),\ \left(\frac{d^2(p_{n+1}, Sp_n)}{1+d(p_{n+1}, Sp_n)}\right)+\right. \\ \left. d(p_{n+1}, Sp_n)d(p_{n+1}, Sp_{n+1}),\ \left(\frac{d^2(p_n, Sp_{n+1})}{1+d(p_n, Sp_n)d(p_{n+1}, Sp_n)}\right)\right] \end{matrix}\right\}$$

$$\max\left\{\begin{matrix} d^2(p_n, p_{n+1}), d(p_n, p_{n+1})d(p_{n+1}, p_{n+1}) \\ \left(\frac{d^2(p_{n+1}, p_{n+1})}{1+d(p_{n+1}, p_{n+1})}\right), d(p_{n+1}, p_{n+1})d(p_n, p_{n+1}), \\ \frac{r}{2s}\left[d(p_n, p_{n+1})d(p_n, p_{n+2}),\ \left(\frac{d^2(p_{n+1}, p_{n+1})}{1+d(p_{n+1}, p_{n+1})}\right)\right. \\ \left. +d(p_{n+1}, p_{n+1})d(p_{n+1}, p_{n+2}),\ \left(\frac{d^2(p_n, p_{n+2})}{1+d(p_n, p_{n+1})d(p_{n+1}, p_{n+1})}\right)\right] \end{matrix}\right\}$$

Now, using triangle inequality of b- metric space, taking limit as $n \to \infty$ and using Inequality (3.1.5) we have

$$\leq \max\left\{d_n^2,\ \frac{r}{2s}\left[d_n\, d(p_n, p_{n+2})\ 0, 0,\right.\right\}$$

$$\log_{n\to\infty} k(p_n, p_{n+1}) \leq \max\left\{k^2,\ \frac{1}{2s}\left[ks\, d(k+k)\ 0, 0,\right.\right\} = k^2 \qquad (3.1.7)$$

Now taking limit as $n \to \infty$ in inequality (3.1.6) and using Inequality (3.1.7) we have

$(1 + rk)\, k^2 \leq \frac{r}{s} k^3 + k^2 - \varphi(k^2) \leq r\, k^3 + k^2 - \varphi(k^2)$

Which gives $\varphi(k^2) \leq 0$, which implies that $k = 0$.

Therefore,

$$\lim_{n\to\infty} d_n = \lim_{n\to\infty} d(p_n, p_{n+1}) = 0 \qquad (3.1.8)$$

Now , we show that $\{p_n\}$ is a cauchy sequence in Z. Let m, n ∈ N,m > n , then $d(p_n, p_m) \leq s\ \{\ d(p_n, p_{n+1}) + d(p_{n+1}, p_m)\ \} \leq s\, d(p_n, p_{n+1}) + s^2\{\ d(p_{n+1}, p_{n+2}) + d(p_{n+2}, p_m)\ \}$

$\leq s\, d(p_n, p_{n+1}) + s^2\, d(p_{n+1}, p_{n+2}) + s^3\, d(p_{n+2}, p_{n+3}) + \ldots\ldots\ldots\ldots$

$= s\, d_n + s^2\, d_{n+1} + s^3\, d_{n+2} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

Taking limit $n \to \infty$ and using Inequality (3.1.8) we get $\lim_{n\to\infty} d(p_n, p_m) = 0$

Hence $\{p_n\}$ is a cauchy sequence in Z. But Z is a complete b- matric space and therefore there exists some p∈ Z

Such that $\lim_{n\to\infty} p_n = p \qquad (3.1.9)$

Now ,we prove that p is a fixed point of S.

Replacing q by p_n in inequality (3.1.1),we obtain

$$[1 + rd(p, p_n)]\, d^2(Sp, Sp_n) \leq \frac{r}{s} \max\ \left\{\frac{1}{2}\ \left[d^2(p, Sp)\, d(q, p_{n+1}) + d(p, Sp)\, d^2(p_n, p_{n+1}),\ \frac{d^2(p_n, Sp)}{1+d(p_n, Sp)}\right],\right.$$

$$\left[d(p, Sp)\, d(p, p_{n+1})\right.$$

$$\left. d(p_n, Sp) + \frac{d^2(p, p_{n+1})}{1+d(p, p_{n+1})(p_n, Sp)}\right], d(p, p_{n+1})\, d(p_n, Sp)\, d(p_n, p_{n+1})\ \} + m(p, p_n) \qquad -\varphi\{m(p, p_n)\}$$

$$[1 + rd(p, p_n)]\, d^2(Sp, p_{n+1}) \leq r \max\ \left\{\frac{1}{2}\ \left[d^2(p, Sp)\, d(q, Sp_n) + d(p, Sp)\, d^2(p_n, Sp_n), \frac{d^2(p_n, Sp)}{1+d(p_n, Sp)}\right],\right.$$

$$\left[d(p, Sp)\, d(p, Sp_n)\, d(p_n, Sp) + \frac{d^2(p, Sp_n)}{1+d(p, Sp_n)(p_n, Sp)}\right], d(p, Sp_n)\, d(p_n, Sp)\, d(p_n, Sp_n)\ \} + m(p, p_n) \qquad -\varphi\{m(p, p_n)\}$$

(3.1.10)

Where

$$k(p, p_n) = \max\left\{\begin{matrix} d^2(p, p_n), d(p, Sp)d(p_n, Sp)\frac{d^2(p_n, Sp)}{1+d(p_n, Sp)}, \\ d(p_n, Sp)d(p, Sp),\ \frac{1}{2s}\left[d(p, Sp)d(p, Sp_n), \left(\frac{d^2(p_n, Sp)}{1+d(p_n, Sp)}\right)+\right. \\ \left. d(p_n, Sp)d(p_n, Sp_n), \left(\frac{d^2(p, Sp_n)}{1+d(p, Sp)(p_n, Sp)}\right)\right] \end{matrix}\right\}$$

$$k(p, p_n) = \max\left\{\begin{matrix} d^2(p, p_n), d(p, Sp)d(p_n, Sp)\frac{d^2(p_n, Sp)}{1+d(p_n, Sp)}, d(p_n, Sp)d(p, Sp), \\ \frac{1}{2s}\left[d(p, Sp)d(p, p_{n+1}), \left(\frac{d^2(p_n, Sp)}{1+d(p_n, Sp)}\right)+\ d(p_n, Sp)d(p_n, Sp_n)\right. \\ \left., \left(\frac{d^2(p, p_{n+1})}{1+d(p, Sp)(p_n, Sp)}\right)\right] \end{matrix}\right\}$$

Now taking limit as $n \to \infty$ and using Inequality (3.1.8) and Inequality (3.1.9) we get

$$\lim_{n\to\infty} k(p, p_n) = \max\left\{0,0,0\ \ \frac{1}{2s}\left[0,0+\ 0,0\right]\right\} = 0 \qquad (3.1.11)$$

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Now taking limit as n→ ∞ and using Inequality (3.1.10) and Inequality (3.1.11) we have

$[1 + 0] d^2(Sp, p) \leq \frac{r}{s} \max \{\frac{1}{2} [0+0, 0], [0+0], 0\} + 0 - \varphi(0) = 0$

This implies that $d^2(Sp, p) \leq 0$. Thus $d^2(Sp, p) = 0$ and consequently, d(Sp,p) = 0

Thus Sp = p

Uniqueness: Let p and q be two fixed points of S.Then by using Sp = p and Sq = q ,inequality(1)

Reduces to

$[1 + rd(p,q)] d^2(p, q) \leq \frac{r}{s} \max \{\frac{1}{2} [d^2(p,p) d(q,q) + d(p,p) d^2(q,q), \frac{d^2(q,p)}{1+ d(q,p)}], [d(p,p) d(p,q) d(q,p) + \frac{d^2(p,q)}{1+ d(p,q)(q,p)}], d(p,q) d(q,p) d(q,q) \} + m(p, q) - \varphi\{m(p, q)\}$ (3.1.12)

Where

$k(p,q)=\max\left\{d^2(p, q), d(p, p)d(q, p)\frac{d^2(q,p)}{1+ d(q,p)}, d(q, p)d(p, p), \frac{1}{2s}[d(p, p)d(p, q), \left(\frac{d^2(q,p)}{1+ d(q,p)}\right) + d(q, p)d(q, q), \left(\frac{d^2(p,q)}{1+ d(p,p)(q,p)}\right)]\right\}$

$k(p,q) = \max \{ d^2(p, q), 0,0, d^2(p, q), 0\}$

$= d^2(p, q)$

So Inequality (3.1.12) becomes

$[1 + r d(p,q)] d^2(p, q) \leq \frac{r}{s} \max \{ 0, 0 , 0\} + d^2(p, q) - \varphi\{d^2(p, q)\}$

$rd^3(p, q) \leq -\varphi\{d^2(p, q)$

$rd^3(p, q) + \varphi d^2(p, q) \leq 0,$

Which is only possible when $d^2(p, q) = 0$.

This implies that p = q .

Hence S has a unique fixed point.

Now we furnish an example that shows the existence of a unique fixed point for a self map in complete b-metric space Z(Z is not a metric space as shown in example 2.3)

Corollary 3.3

Let (Z, d, s) be a complete b- metric space and S be a self –map of Z satisfying

$d^2(Sp,Sq) \leq m(p,q) - \varphi(m(p, q))$,

where $M(p, q) = \max\{d^2(p,q), d(p, Sp) d(q, Sq), d(p, Sq)d(q, Sp), \frac{1}{2s}[d(p,Sq)d(p,Sq)+ d(q,Sp)d(q,Sq)]\}$

$R \geq 0$ is a real number and $\varphi : [0,\infty) \to [0, \infty)$ is a continuous function with $\varphi(t) = 0$ if and only if t = 0 and $\varphi(t) > 0$ for each t > 0. Then S has a unique fixed points in Z.

Now we extend theorem 3.1 by finding a common fixed points result for two self maps using generalized φ – weak cotractive condition with cubic term involving r in the setting of b-metric space.

Theorem 3.2 : Let (Z, d, s) be a complete b- metric space and L, M self maps of Z such that $M(Z) \subset L(Z)$ and the pair {L,M} is weakly compatible. Also,

$[1 + rd(Lp,Lq)] d^2(Mp, Mq) \leq \frac{r}{s} \max \{\frac{1}{2} [d^2(Lp, Lp) d(Lq, Mq) + d(Lp, Mp) d^2(Lq, Mq), \frac{d^2(Lq,Mp)}{1+ d(Lq,Mp)}], [d(Lp,Mp) d(Lp, Mq) d(Lq, Mp) + \frac{d^2(Lp,Mq)}{1+ d(Lp,Mq)(Lq,Mp)}], d(Lp, Mq) d(Lq, Mp) d(Lq, Mq) \} + m(Lp, Lq) - \varphi\{m(Lp, Lq)\}$ (3.2.1)

Where

$$k(Lp, Lq) = \max\left\{\begin{array}{c} d^2(Lp, Lq), d(Lp, Mp)d(Lq, Mp)\frac{d^2(Lq,Mp)}{1+ d(Lq,Mp)}, \\ d(Lq, Mp)d(Lp, Mp), \frac{1}{2s}[d(Lp, Mp)d(Lp, Mq), \left(\frac{d^2(Lq,Mp)}{1+ d(Lq,Mp)}\right) + \\ d(Lq, Mp)d(Lq, Mq), \left(\frac{d^2(Lp,Mq)}{1+ d(Lp,Mp)(Lq,Mp)}\right)] \end{array}\right\},$$

$r \geq 0$ is areal number and and $\varphi : [0,\infty) \to [0, \infty)$ is a continuous function with $\varphi(t) = 0$ if and only if t = 0 and $\varphi(t) > 0$ for each t > 0. Then L and M have a unique fixed points in Z.

Proof : Let p_0 be an arbitrary point in Z. Since $M(Z) \subset L(Z)$ .Therefore we can find some p_1 such that $Mp_0 = Lp_1 = q_1$.

We define a sequence $\{q_n\}$ in Z by $q_n = Mp_{n-1} = L p_n$, $n \geq 0$. (3.2.2)

For convenience, we write $d(q_n, q_{n+1}) = d_n$ (3.2.3)

First we show that $\{q_n\}$ is a monotonically decreasing sequence, i.e. $d_{n+1} \leq d_n$

Case 1: If n is odd. Take n = 2z-1 .We prove that $d_{2z} \leq d_{2z-1}$

Now taking $p = p_{2z-1}$, $q = q_{2z}$ in Inequality(3.2.1),we have

$[1 + rd(Lp_{2z-1},Lq_{2z})] d^2(Mp_{2z-1}, Mq_{2z}) \leq \frac{r}{s} \max \{\frac{1}{2} [d^2(Lp_{2z-1},Lp_{2z-1}) d(Lq_{2z},Mq_{2z}) + d(Lp_{2z-1},Mp_{2z-1}) d^2(Lq_{2z},Mq_{2z}), \frac{d^2(Lq_{2z},Mp_{2z-1})}{1+ d(Lq_{2z},Mp_{2z-1})}], [d(Lp_{2z-1},Mp_{2z-1}) d(Lp_{2z-1},Mq_{2z}) d(Lq_{2z},Mp_{2z-1}) + \frac{d^2(Lp_{2z-1},Mq_{2z})}{1+ d(Lp_{2z-1},Mq_{2z})(Lq_{2z},Mp_{2z-1})}], d(Lp_{2z-1},Mq_{2z})$

$d(Lq_{2z},Mp_{2z-1})\ d(Lq_{2z},Mq_{2z})\ \} + m\ (Lp_{2z-1},Lq_{2z}) - \varphi\{m(Lp_{2z-1}, L\ q_{2z})\}$

Where

$k(Lp_{2z-1},Lq_{2z})=$

$$\max\left\{\begin{array}{c} d^2(Lp_{2z-1},Lq_{2z}), d(Lp_{2z-1},Mp_{2z-1})d(Lq_{2z},Mp_{2z-1}) \\ \frac{d^2\ (Lq_{2z},Mp_{2z-1})}{1+\ d(Lq_{2z},Mp_{2z-1})}\ ,\ d(Lq_{2z},Mp_{2z-1})d(Lp_{2z-1},Mp_{2z-1}), \\ \frac{1}{2s}\ [d(Lp_{2z-1},Mp_{2z-1}) \\ d(Lp_{2z-1},Mq_{2z}), \left(\frac{d^2\ (Lq_{2z},Mp_{2z-1})}{1+\ d(Lq_{2z},Mp_{2z-1})}\right) +\ d(Lq_{2z},Mp_{2z-1}) \\ d(Lq_{2z},Mq_{2z}), \left(\frac{d^2\ (Lp_{2z-1},Mq_{2z})}{1+\ d(Lp_{2z-1},Mp_{2z-1})(Lq_{2z},Mp_{2z-1})}\right)] \end{array}\right\}$$

Now using Inequality (3.2.2) ,we get

$\lceil 1 + rd(p_{2z-1},q_{2z})\rceil\ d^2\ (p_{2z},q_{2z+1}) \le \frac{r}{s}\max\ \{\frac{1}{2}\ [d^2\ (p_{2z-1},p_{2z-1})\ d(q_{2z},q_{2z+1}) + d(p_{2z-1},p_{2z})\ d^2(p_{2z},q_{2z+1})\ ,\frac{d^2\ (q_{2z},p_{2z})}{1+\ d(q_{2z},p_{2z})}\ \rceil,\lceil$
$d(p_{2z-1},p_{2z})\ d\ (p_{2z-1},q_{2z+1})\ d(q_{2z},p_{2z}) + \frac{d^2\ (p_{2z-1},q_{2z+1})}{1+\ d(p_{2z-1},q_{2z+1})(q_{2z},p_{2z})}\ \rceil,\ d(p_{2z-1},q_{2z+1})\ d(q_{2z},p_{2z})\ d(q_{2z},q_{2z+1})\ \} + m\ (p_{2z-1},q_{2z}) - \varphi\{m(p_{2z-1}\ , q_{2z})\}$

Where

$$k(Lp_{2z-1},Lq_{2z})= \max\left\{\begin{array}{c} d^2(p_{2z-1},q_{2z}), d(p_{2z-1},p_{2z})d(q_{2z},p_{2z})\frac{d^2\ (q_{2z},p_{2z})}{1+\ d(q_{2z},Mp_{2z})}\ , \\ d(q_{2z},p_{2z})d(p_{2z-1},p_{2z})\ ,\ \frac{1}{2s}\ [d(p_{2z-1},p_{2z})d(p_{2z-1},q_{2z+1}), \\ \left(\frac{d^2\ (q_{2z},p_{2z})}{1+\ d(q_{2z},p_{2z})}\right) +\ d(q_{2z},p_{2z})d(q_{2z},q_{2z+1}), \left(\frac{d^2\ (p_{2z-1},q_{2z+1})}{1+\ d(p_{2z-1},p_{2z})(q_{2z},p_{2z})}\right)] \end{array}\right\}$$

Then

$\lceil 1 + rd_{2z-1}]d_{2z}^2 \le \frac{r}{s}\max\ \{\frac{1}{2}\ [0 + d_{2z-1}\ \ d_{2z}^2,\ \ 0\rceil,\lceil 0 + 0\ \rceil,0\ \} + m\ (p_{2z-1},q_{2z}) - \varphi\{m(p_{2z-1}\ , q_{2z})\}$ (3.2.4)

Where

$k(Lp_{2z-1},Lq_{2z}) =\max\left\{d_{2z-1}^2, 0\ ,\ 0,\ \frac{1}{2s}\ [d_{2z-1}\ s(d_{2z-1}\ +\ d_{2z}\ ),0,0]\right\}$

$\lceil$By triangle Inequality of b- metric space $\rceil$

If $d_{2z-1} < d_{2z}$

Then

$M(p_{2z-1}\ ,d_{2z}) < \max\ \{d_{2z}^2, d_{2z}^2\ , 0\ d_{2z}^2\} = d_{2z}^2$

So Inequality(3.2.3) becomes

$d_{2z}^2 + rd_{2z-1}d_{2z}^2 \le \frac{r}{s}\max\ \{\frac{1}{2}\ [d_{2z-1}\ \ d_{2z}^2,\ \ 0,0,0\ \} - \varphi\{d_{2z}^2\ \}$

$\le r\ rd_{2z-1}d_{2z}^2 + d_{2z}^2\ - \varphi\{d_{2z}^2\ \}$

$\lceil d_{2z-1}^2 < d_{2z}^2$,gives $- \varphi\{d_{2z}^2\ \} < - \varphi\{d_{2z-1}^2\}\rceil$

Which gives $d_{2z} \le d_{2z-1}$

Case 2: If n is a even, taking $n = 2z$ and putting $p = p_{2z}, q = d_{2z+1}$ in Inequality (3.1.1) and proceeding as above we can easily obtain that $d_{2z+1} \le d_{2z}$

Therefore $\{d_z\}$ Is a monotonically decreasing sequence of nonnegative real number. Now proceding as in 3.1 we can easily prove that

$\lim_{n\to\infty} d_z = \lim_{n\to\infty} d(q_z, q_{z+1}\ ) = 0$ and $\{q_z\}$ is a Cauchy sequence in Z.

But Z is a complete b- metric space and therefore exists some $q \in Z$ such that

$\lim_{n\to\infty} q_z = q$ (3.2.5)

Since $q \in M(Z) \subset L(Z)$, therefore,there exist a point $v \in Z$ such that $q = Lv$. Now we prove that $Mv = q$. For this replacing p by v and q by p_z in inequality (3.2.1), we obtain

$+ rd(Lv,\ Lp_z)\rceil\ d^2\ (Mv, Mp_z) \le \frac{r}{s}\max\ \{\frac{1}{2}\ [d^2(Lv,Lv)\ d(Lp_z,Mp_z) + d(Lv,Mv)\ d^2(Lp_z,Mp_z)\ ,\frac{d^2\ (Lp_z,Mv)}{1+\ d(Lp_z,Mv)}\ \rceil,\lceil d(Lv,\ Mv)\ d(Lv,$
$Mp_z)\ d(Lp_z,Mv) + \frac{d^2\ (Lv,Mp_z)}{1+\ d(Lv,Mp_z)(Lp_z,Mv)}\ \rceil,\ d(Lv,Mp_z)\ d(Lp_z,Mv)\ d(Lp_z,Mp_z)\}+m(Lv,Lp_z)-\varphi\{m(Lv\ , L\ p_z)\}$ (3.2.6)

Where

$k(Lv,Lp_z)=$

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$$\max\left\{\begin{array}{c} d^2(Lv, Lp_z), d(Lv, Mv)d(Lp_z, Mv)\frac{d^2(Lp_z,Mv)}{1+d(Lp_z,Mv)}, d(Lp_z, Mv)d(Lv, Mv), \\ \frac{1}{2s}[d(Lv, Mv)d(Lv, Mp_z), \left(\frac{d^2(Lp_z,Mv)}{1+d(Lp_z,Mv)}\right) + d(Lp_z, Mv)d(Lp_z, Mp_z), \\ \left(\frac{d^2(Lv,Mp_z)}{1+d(Lv,Mv)(Lp_z,Mv)}\right)] \end{array}\right\}$$

$[1 + rd(q,q_z)]\, d^2(Mv, q_{z+1}) \leq \frac{r}{s}\max\{\frac{1}{2}[d^2(q,q)\, d(q_z,q_{z+1}) + d(q,Mv)\, d^2(q_z,q_{z+1}), \frac{d^2(q_z,Mv)}{1+d(q_z,Mv)}], [d(q,Mv)\, d(q,q_{z+1})\, d(q_z,Mv) + \frac{d^2(q,q_{z+1})}{1+d(q,q_{z+1})(q_z,Mv)}], d(q,q_{z+1})\, d(q_z,Mv)\, d(q_z,q_{z+1})\} + m(q,q_z) - \varphi\{m(q, q_z)\}$

Where

$$k(q,q_z) = \max\left\{\begin{array}{c} d^2(q, q_z), d(q, Mv)d(q_z, Mv)\frac{d^2(q_z,Mv)}{1+d(q_z,Mv)}, \\ d(q_z, Mv)d(q, Mv), \frac{1}{2s}[d(q, Mv)d(q, q_{z+1}), \\ \left(\frac{d^2(q_z,Mv)}{1+d(q_z,Mv)}\right) + d(q_z, Mv)d(q_z, q_{z+1}), \left(\frac{d^2(q,q_{z+1})}{1+d(q,Mv)(q_z,Mv)}\right)] \end{array}\right\}$$

Now taking limit as $n \to \infty$ and using Inequality (3.2.5), we get

$d^2(Mv, q) \leq \frac{r}{s}\max\{0,0,0\} + m(q,q_z) - \varphi\{m(q, q_z)\}$

where

$K(q, q_z) = \max\{0,0,0,0,0\} = 0$

Therefore, $K(q, q_z) \leq 0$, which gives $q = Mv$

Hence $Lv = q = Mv$. Thus v is coincidence point of L and M. Now since {L,M}is weakly compatible, therefore $LMv = MLv$

Which gives $Lq = Mq$.

Now we claim that q is common fixed point of L and M. For this we replace p by v in Inequality(3.2.1)

$[1 + rd(Lv, Lq)]\, d^2(Mv, Mq) \leq \frac{r}{s}\max\{\frac{1}{2}[d^2(Lv,Lv)\, d(Lq,Mq) + d(Lv,Mv)\, d^2(Lq,Mq), \frac{d^2(Lq,Mv)}{1+d(Lq,Mv)}],$

$[d(Lv,Mv)\, d(Lv, Mq)\, d(Lq, Mv) + \frac{d^2(Lv,Mq)}{1+d(Lv,Mq)(Lq,Mv)}], d(Lv,Mq)\, d(Lq,Mv)\, d(Lq,Mq)\} + m(Lv,Lq) - \varphi\{m(Lv, Lq)\}$

Where

$$k(Lv,Lq) = \max\left\{\begin{array}{c} d^2(Lv, Lq), d(Lv, Mv)d(Lq, Mv)\frac{d^2(Lq,Mv)}{1+d(Lq,Mv)}, \\ d(Lq, Mv)d(Lv, Mv), \frac{1}{2s}[d(Lv, Mv)d(Lv, Mq), \\ \left(\frac{d^2(Lq,Mv)}{1+d(Lq,Mv)}\right) + d(Lq, Mv)d(Lq, Mq), \left(\frac{d^2(Lv,Mq)}{1+d(Lv,Mp)(Lq,Mv)}\right)] \end{array}\right\}$$

Since $Lv = q = Mv$ and $Mq = Lq$,Therefore

$[1 + rd(q, Lq)]\, d^2(q, Lq) \leq \frac{r}{s}\max\{\frac{1}{2}[d^2(q,q)\, d(Lq, Lq) + d(q, q)\, d^2(Lq, Lq), \frac{d^2(Lq,q)}{1+d(Lq,q)}], [d(q,q)\, d(q,Lq)\, d(Lq,q) + \frac{d^2(q,Lq)}{1+d(q,Lq)(Lq,q)}], d(q, Lq)\, d(Lq, q)\, d(Lq, Lq)\} + m(q,Lq) - \varphi\{m(q, Lq)\}$

Where

$$k(q, Lq) = \max\left\{d^2(q, Lq), d(q, q)d(Lq, q)\frac{d^2(Lq,q)}{1+d(Lq,q)}, d(Lq, q)d(q, q), \frac{1}{2s}[d(q, q)d(q, Lq), \left(\frac{d^2(Lq,q)}{1+d(Lq,q)}\right) + d(Lq, q)d(Lq, Lq), \left(\frac{d^2(q,Lq)}{1+d(q,Lq)(Lq,q)}\right)]\right\}$$

$k(q,Lq) = \max\left\{d^2(q, Lq), 0, 0, \frac{1}{2s}[0,0 + 0,0]\right\} = d^2(q, Lq)$

Therefore,

$d^2(q, Lq) + r\, d^3(q, Lq) \leq \frac{r}{s}.0 + d^2(q, Lq) - \varphi\{d^2(q, Lq)\}$.

This implies that

$rd^3(q, Lq) + \varphi\{d^2(q, Lq)\} \leq 0$, which gives $d(q,Lq) = 0$ and hence $q = Lq = Mq$.

Thus q is common fixed point of L and M.

Uniqueness: Let c be a another common fixed point of L and M .Then using Inequality (3.2.1) , we have

$[1 + rd(Lc, Lq)]\, d^2(Mc, Mq) \leq \frac{r}{s}\max\{\frac{1}{2}[d^2(Lc,Lc)\, d(Lq,Mq) + d(Lc,Mp)\, d^2(Lq,Mq), \frac{d^2(Lq,Mc)}{1+d(Lq,Mc)}], [d(Lc,Mc)\, d(Lc,Mq)\, d(Lq,Mc) + \frac{d^2(Lc,Mq)}{1+d(Lc,Mq)(Lq,Mc)}], d(Lc,Mq)\, d(Lq,Mc)\, d(Lq,Mq)\} + m(Lc,Lq) - \varphi\{m(Lc, Lq)\}$ (3.2.7)

Where

$k(Lc,Lq)= \max\left\{d^2(Lc,Lq), d(Lc,Mc)d(Lq,Mc)\frac{d^2(Lq,Mc)}{1+d(Lq,Mc)}, d(Lq,Mc)d(Lc,Mc), \frac{1}{2s}[d(Lc,Mc)d(Lc,Mq), \left(\frac{d^2(Lq,Mc)}{1+d(Lq,Mc)}\right) + d(Lq,Mc)d(Lq,Mq), \left(\frac{d^2(Lc,Mq)}{1+d(Lc,Mp)(Lq,Mc)}\right)]\right\}$

$[1 + rd(c,q)]\, d^2(c,q) \leq \frac{r}{s}\max\{\frac{1}{2}[d^2(c,c)\,d(q,q) + d(c,p)\,d^2(q,q), \frac{d^2(q,c)}{1+d(Lq,Mc)}], [d(c,c)\,d(Lc,q)\,d(q,c) + \frac{d^2(c,q)}{1+d(c,q)(q,c)}], d(c,q)\,d(q,c)\,d(q,q)\} + m(c,q) - \varphi\{m(c,q)\}$ (3.2.8)

$k(c,q)= \max\left\{\begin{matrix} d^2(c,q), d(c,c)d(q,c)\frac{d^2(q,c)}{1+d(q,c)}, d(q,c)d(c,c), \\ \frac{1}{2s}[d(c,c)d(c,q), \left(\frac{d^2(q,c)}{1+d(q,c)}\right) + d(q,c)d(q,q), \left(\frac{d^2(c,q)}{1+d(c,c)(q,c)}\right)] \end{matrix}\right\}$

$= d^2(c,q)$

Solving inequality (18),we have c = q.

Consequently ,q is a unique common fixed point of L and M.

Now ,we given example in support of our theorem.

Conclusion:

In the present paper, firstly ,we have established a fixed point theorem using a generalized φ weak contractive condition involving cubic terms of d(p,q) in the setting of b metric space and then we have generalized the results by finding a unique common fixed point of two self mappings. The results proved in this paper generalized and extend various known existing results. Further, some suitable examples are also given which show the generality of our proven results over the existing ones.

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References:

1. Bakhtin IA.The contraction mapping principle in almost metric spaces. Functional Analysis.(1989);30:26–37.
2. Czerwik S. Contraction mappings in b-metric spaces. Act a Mathematical Informatics Universitatis Ostraviensis.(1993); 1:5-1.https://dml.cz/bitstream/handle/10338.dmlcz/120469/ActaOstrav_01-1993-1_2.pdf.
3. Aydi H, Bota M, Karapinar E, Moradi S.A common fixed point for weak ϕ-contractions on b-metric spaces. Fixed Point Theory.(2012);13(2):337346
4. A k kouchi M. Common fixed point theorems for two self-mappings of a b-metric space under an implicit relation. Hacettepe Journal of Mathematics and Statistics.(2011);40(6):805–810.
5. Kir M, Kiziltunc H.On some well known fixed point theorems in b-metric spaces. Turkish Journal of Analysis and Number Theory.(2013);1(1):13–16.
6. Agrawal S, Qureshi K, Nema J.A Fixed Point Theorem for b-Metric Space. International Journal of Pureand Applied Mathematical Sciences. 2016;9(1):45–50 Availablefrom:https://www.ripublication.com/ijpams16/ijpamsv9n1_05.pdf.
7. HaoY,Guan H.Common Fixed Point Theorems in b-Metric Spaces. Turkish Journal of Analysis and Number Theory. (2019);7(4):117-123.Available from:https://pubs.sciepub.com/tjant/7/4/4/index.html.
8. Jain K, Kaur J. Some Fixed Point Results in b-Metric Spaces and b-Metric-Like Spaces with New Contractive Mappings. Axioms. 2021;10(2):
9. Guan H, Li J. Common Fixed-Point Theorems of Generalized (ψ,)-Weakly Contractive Mappings in b-Metric-Like Spaces and Application. Journal of Mathematics.(2021):1–4.
10. Petru¸sel A, Petru¸sel J.Graphical contractions and common fixed points in b-metric spaces. Arabian Journal of Mathematics.2023;12:423–430.Availablefrom:https://link.springer.com/article/10.1007/s40065-022-00396-8.
11. Peng W, HeF, Liu X. Answers to questions on Kannan's fixed point theorem in strong b-metric spaces. AIMS Mathematics.2024;9 (2):3671–3684.
12. Murthy P P, Prasad KNVVV.We ak contraction condition involving cubic terms of d(x, y) under the fixed point consideration. Journal of Mathematics. (2013);:1–5.Available from: https://doi.org/10.1155/2013/967045.
13. Dutta P N, Choudhury B S. A generalizati on of contraction principle in metric spaces.Fixed Point Theory and Applications.(2008):1–8.Availablefrom:https://doi.org/10.1155/2008/406368

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Advances in Data Security in Cryptography

Mohit Shrivastava[1], Kanchan Mishra[2,*]

Abstract

In the ultra-modern period, evaluation of networking and wireless networks within information and communication technology has brought many changes to deal with this technology using internet, growing strongly over the past several decades, data security has come a main concern for anyone connected to the web. Data security ensures that our data can only be accessed by authorized recipients and prevents any unauthorized access or alteration of the data. We punctuate the strengths and sins of state-of-the-art ways, and conclude that, while no single approach is likely to approach as a tableware pellet. Therefore, the key is to combine different tools and software approaches to work in confluence using partitioned computing, where a computation is resolved across different cryptographic ways precisely, so as not to compromise security. We punctuate some recent work in that direction. The significant volume, rapid pace, and diverse nature of this data pose a security risk. Given the constraints of IoT devices such as low power and limited computational speed, traditional encryption methods like DES, 3DES, and AES are too complex for implementation. Therefore, there is a necessity to create lightweight encryption algorithms tailored for IoT devices to ensure secure communication and data transmission within IoT environments. Cryptographic and steganographic techniques are employed to safeguard data transmitted over the internet.

Keywords: Cryptography, security, encryption, decryption, big data

INTRODUCTION

With cloud emerging as the dominant computing platform, secure data outsourcing, originally described in significant data management challenge [1–10]. Initial work on secure computing focused extensively on encryption techniques to allow operations to execute on the cloud. With the very fast growth of internet technology, our lives are steadily led into a virtual world. We are going to live in an imaginary space.

In the virtual realm, individuals have the freedom to engage in various activities such as shopping, chatting, and working. Internet of things is an emerging domain which will be the main reason for generating the Big Data. Big Data is also an emerging field and the latest topic of interest. IoT devices are increasing day by day, and the data being generated by them also increases as shown in Figure 1.

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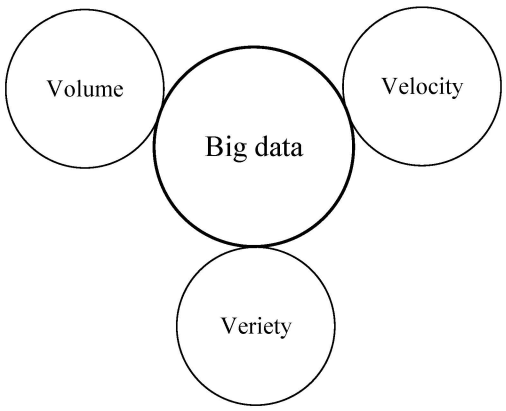


Figure 1. Big data flow graph.

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LITERATURE REVIEW

1. *Saini and Susan [11]:* It has been observed that network and computer security represent a rapidly evolving field within computer science, where the pursuit of secure computing is an ongoing challenge. Courses in computer security primarily emphasize on algorithmic and mathematical elements such as hashing methods and encryption. As hackers devise new ways to breach network systems, educational programs adapt by introducing courses that address the latest attack methods. However, these attack techniques quickly become obsolete as new security software respond to them. The terminology and techniques in security are continually evolving, leading to the emergence of new skills and practices in various domains such as business operations, network optimization, security architecture, and legal frameworks.
2. In a study concerning network security and cryptography, Tayal highlighted the increasing volume of data generated daily by global organizations due to the rise of social networks and e-commerce applications [12]. Consequently, ensuring secure data transfer over the internet has become a significant concern. The growing number of internet users underscores the importance of employing cryptography methods to address this issue. The paper offers a summary of different approaches employed by networks to bolster security, including cryptography.
3. *Qadir and Varol [13]:* The research focused on symmetric encryption, utilized for encrypting specific text or speech. In this investigation, the data intended for encryption undergoes initial conversion into an encapsulated cipher format, rendering it indecipherable by a cipher algorithm.
4. *Chachapara and Bhadlawala [14]:* They explored secure data sharing through cryptography in cloud computing, presenting a framework that employs cryptographic algorithms such as RSA and AES. Among these algorithms, AES is identified as the most robust cryptographic method. Within this framework, cloud users have the capability to generate keys tailored for various users, each with different levels of permissions for accessing their files.
5. *Orman [15]:* The author noted that extensive debates and advancements are ongoing in the field of cryptography. They emphasized the crucial role of hash functions in cryptography, which assign a unique numerical value to each piece of data. As vulnerabilities in MD5 became apparent over time, it sparked uncertainty regarding the appropriate design of hash functions.
6. *Gennaro et al. [16]:* They explored the concept of randomness in cryptography, highlighting that a random process is characterized by unpredictable outcomes. The discussion emphasized the significance of randomness in cryptography as it enables the generation of information that remains inaccessible to adversaries, preventing them from learning or predicting it.
7. *Preneel et al. [17]:* The authors showcased the application of cryptography and information security in the aftermath of the Snowden revelations, addressing issues such as mass surveillance and the vulnerability of ICT systems. Additionally, they examined various methods through which advanced attackers can circumvent or weaken cryptographic measures.

BACKGROUND OF SECURITY TECHNIQUES

Steganography

"Steganography derived from two Greek words 'steganos' which means either secret or covered and 'graphein' which means writing or drawing". Which shows that steganography means secret notation, secret delineation, covered notation, or covered delineation. About 2000 times agone, the Greek used this fashion to transfer secret dispatches [18–20]. Steganography means hiding a secret communication within another communication. Steganography is used to hide secret data into the non-secret data. The train in which the secret data is concealed is called the carrier. After concealing the secret data, the modified carrier looks like an original carrier. Some swish carriers are images, audio and video lines. In steganography, we do not reckon the data, we simply conceal our data within image, audio or video lines.

Cryptography

Cryptography is used to make security. Cryptography is an important tool to cover the data. Our computer word is secured by cryptographic hash function. When we shoot a dispatch, it is also secured by cryptography fashion SSL [20–30].

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Cryptography concerns with confidentiality (No one can pierce the data except the person to whom it may concern), integrity (No one can change or modify the data), and authenticate (sender and receiver conform their identity). Cryptography is used to store and transfer the data in such a form that only sender and receiver can understand it or reuse it. A meddler cannot pierce or understand that data. Cryptography depends upon the algorithm and the key. Two main terms used in cryptography are encryption and decryption. Encryption is a process to convert a plain textbook into cipher textbook and decryption is the process to convert cipher textbook to plain textbook [31–40].

Featherlight Cryptography in Internet of Effects machines connected to machines or machines connected to mortal to communicate or to shoot data. IoT bias have lower computational power, so we need to design featherlight encryption ways to cipher the data. Featherlight Cryptography is used in a constrained terrain like RFID markers, healthcare bias, and detectors. While developing Lightweight Cryptography ways, we must have to take into consideration the software and tackle specification.

CRYPTOGRAPHIC SECURE DATA PROCESSING

Inimical and security models for the Cloud: Cryptographic ways are designed to deal with different types of inimical public shadows, videlicet honest-but-curious (HBC) and vicious shadows. An HBC inimical pall, which is considered extensively in numerous cryptographic algorithms, stores an outsourced dataset without tampering, rightly computes assigned tasks, and returns answers; still, it may exploit lateral knowledge.

To acquire significant insights into sensitive data, investigators focus on querying prosecution, background information, and the scale of the affair. A vicious adversary may diverge from the algorithm and may execute a task that he/she wishes (e.g., delete tuples from the relation). To ensure security in HBC and vicious inimical models, IND-CKA and IND-CKA2 are star security parcels, independently. Further, to deal with securely streamlining and fitting data, forward and backward security parcels were introduced. In this study, we discuss different inimical models and security parcels. Secret-Participating-grounded Data Outsourcing Secret-Participating was constructed singly. In using secret-sharing, the database (DB) proprietor divides a secret value into different fractions, called shares, and sends each share to a set of non-communicating actors/waiters. These waiters cannot know the secret value until they collect shares. Note that the threshold value c is often set to be greater than c to accommodate potentially malicious adversaries which may attempt to alter the value of their shares. We discuss fresh hypotheticals about the inimical model when using secret-sharing, e.g., the adversary cannot machinate with all (or conceivably the maturity of) the waiters. Also, the adversary cannot overhear on a maturity of communication channels between the DB proprietor and the waiters. Note that if the adversary could either machinate with or successfully overhear on the communication channels between the maturity of waiters and the DB proprietor, the secret-sharing fashion will not apply [41–50].

Order-conserving secret-sharing (OP-SS), maintains the order of the values in secret-shares too. Due to maintaining the ordering of values, chancing records with outside/minimum values using OP-SS is trivial, while revealing ordering information to the adversary. In 2006, [37] introduced the first work for data outsourcing using SSS and OP-SS for executing sum, outside, and minimal queries. Another paper by [38]. using OP-SS for aggregation queries requires the DB proprietor to retain each polynomial, which was used to produce database shares. The similar approach, suggesting Secret Sharing Scheme (SSS)-based sum and average queries; still, they bear the DB proprietor to retain tuple-ids of qualifying tuples introduced a new approach for searching over the secret-participated data without taking help from the DB proprietor, utilizing the string-matching operation over the shares at the garçon by applying a MapReduce job. In short, these results offer a limited form of selection or aggregation queries, but overload the DB proprietor (by storing enough data related to polynomials and completely sharing in a query prosecution), are insecure due to OP-SS, or reveal access-patterns. Recent work, OBSCURE eliminates all similar limitations and provides a completely secure and effective result

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for enforcing aggregation queries with selections. OBSCURE exploits OP-SS, while OP-SS is not secure (it is prone to background knowledge attacks, for case) [51, 52].

The way OBSCURE cleverly uses OP-SS, it prevents similar attacks by meetly partitioning data, while still being suitable to exploit OP-SS for effectiveness. SMCQL and Conclave are two systems that allow executing SQL queries among different DB possessors, while pushing most corridor of the calculation in cleartext. Recent performances of Intel CPUs introduced SGX, a collection of microarchitectural mechanisms aimed to cover third-party pall operations from the software mound of an untrusted system. SGX allows us to produce a small, trusted prosecution terrain that is insulated and defended from the rest of the system. For illustration, in the pall, SGX protects the calculation from the operating system controlled by the tenant, but vulnerable to multitudinous operations, system level attacks, and the hypervisor controlled by the pall. In addition to guarding against software attacks, SGX provides encryption of enclave's memory having law and data and integrity is defended by the CPU, when the data leaves the last position of the caching scale. This protects SGX operations from tackle attacks like memory poking. While running on the waiters controlled by the call, from a trust point of view, SGX enclaves are effectively controlled by the customer. 1799 Failing of the being SGX armature. Unfortunately, being executions of SGX are prone to a range of side-channel attacks that exploit one of the microarchitectural factors of the CPU, e.g., branch target buffers pattern-history table caches, DRAM row buffer, runner-tables, runner-fault exceptions, instructors, and academic prosecution capabilities to exfiltrate sensitive data and occasionally the entire memory of the enclave. Runner-fault attacks calculate on the operating system that is under control of the bushwhacker to exfiltrate sensitive data from the enclave by driving runner-faults and tracking enclave's memory accesses at the granularity of memory runners. Branch shadowing attacks allow reconstructing control inflow inside the enclave, and hence, secret data by covering all taken branches through a side-channel in the branch vaticination unit. Cache based side-channel attacks allow bushwhackers to trace all memory accesses within the enclave. Foreshadow and Meltdown-type attacks on SGX allow complete access to enclave's memory and registers. Unexpectedly, the side-channel attacks allow a bushwhacker to reconstruct a significant bit of the sensitive dataset, frequently gaining complete access to it in cleartext [53–59].

To illustrate the power of side-channel attacks, we bandy several recent case studies that is cryptographic keys, graphical images, and sensitive genomic data from the enclave in a practical and realistic script). Data Processing using SGX. Several recent systems surfaced to give secure data processing in the SGX terrain. We give a literal perspective on the problem starting with TrustedDB and Cipherbase, the systems that were first to use secure tackle and initiated the field of secure tackle-grounded data processing. We also bandy recent systems, similar as Opaque Enclave, DBS, Health DB, M2R, and VC3, that are erected to influence SGX. We give an overview of types of queries these systems support, and bandy their advantages and limitations, and whether they give practical security in the face of important side-channel attacks. We also bandy an ongoing microarchitectural work aimed to address side-channel attacks in the coming generation of chips, and possible limitations of tackle approaches. Eventually, we give an overview of possible system-position defenses and algorithmic approaches, furnishing a secure terrain with SGX. Data Partitioning-grounded outsourcing being a secure ways do not gauge to large-sized datasets. For illustration, on TPC-H Line-Item table, executing a simple selection query took 1051 sec on 1 M rows using secret-sharing-grounded Jana and 89 sec using SGX based Opaque on 6 M rows. Partitioned computing involves segregating data into sensitive and non-sensitive categories, offering potential advantages by (i) eliminating costly cryptographic operations on non-sensitive data, and (ii) enabling query processing on non-sensitive data to utilize indicators. Similar indicators (that cannot be fluently supported alongside encryption-grounded mechanisms in a non-interactive setting) are a crucial medium for effective query processing in traditional database systems. Partitioned computing over sensitive/non-sensitive data has been considered, especially, in cold-blooded shadows (e.g., Hybr Ex Sedic Prometheus Tagged-Map Reduce SEMROD and where sensitive data stays at a private pall and only clear text non-sensitive data stays at

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a public pall still, do not generalize to partitioned computing in the public pall setting (where sensitive data is stored cryptographically secure and non-sensitive data resides in clear text). Recent work generalizes the partitioned computing approach at the pall. We bandy a new security challenge due to contemporaneous prosecution of queries on the translated (sensitive) dataset and on the plaintext (non-sensitive) datasets, and also, show the need to new security description, called partitioned data security. We punctuate the significance of partitioned calculation at the pall by illustrating selection query prosecution.

GOAL OF THE TUTORIAL

Outgrowth, intended followership, and duration. This study provides a check on data security and sequestration by preamble reducing the state-of-the-art results from the security literature (especially, secret-sharing-, secure tackle-, and data partitioning-grounded ways) that are particularly applicable for databases. Experimenters, scholars, inventors, interpreters interested in data security and sequestration should be served. We covered content for different cults, as 10 freshman, 40 intermediate, 50 advanced. The duration was 3 h. We show that the being data outsourcing ways and systems are not enough if we aim for systems that operate concurrently.

1. Are effective and general enough (i.e., support significant corridor of SQL) to be practical.
2. Offer sustainable security from the stoner's perspective. Below, we give open questions in different directions. We may suppose about how SSS-grounded results can be developed for large-sized datasets, how they can support complex SQL queries, and how one can minimize the number of shadows while doing all operations at the pall. In the face of multitudinous side channel attacks, naive relinquishment of SGX does not give any practical protection. Some of the challenges will be answered by unborn tackle but some will bear algorithmic and compiler-position results to support effective memory and branch-unconscious calculations. To make partition computing further practical, we need to find ways to execute queries (e.g., join and nested queries) using partitioned computing. Another open problem is in changing how one can classify the data into sensitive and non-sensitive data. Another open problem could be in finding which types of cryptographic ways can be supported by partition computing.

CONCULSION

Data security plays a crucial role in safeguarding digital information from unauthorized access, alteration, or deletion. As the volume of sensitive data stored and transmitted online continues to grow, the significance of data security has escalated. The challenges and advantages associated with technology in terms of data security and privacy are closely intertwined. While technology introduces new threats and vulnerabilities, it also offers contemporary solutions. Through adopting effective strategies, staying abreast of emerging technologies, and implementing robust security protocols, we can ensure the resilient protection of data and privacy in our increasingly digitalized environment. As technology progresses, collaboration among individuals, organizations, and policymakers is essential to address challenges and capitalize on opportunities effectively.

REFERENCES

1. Hacigümüş H, Iyer B, Li C, Mehrotra S. Executing SQL over encrypted data in the database-service-provider model. In Proceedings of the 2002 ACM SIGMOD international conference on Management of data. 2002 Jun 3; 216–227.
2. Agrawal R, Kiernan J, Srikant R, Xu Y. Order preserving encryption for numeric data. In Proceedings of the 2004 ACM SIGMOD international conference on Management of data. 2004 Jun 13; 563–574.
3. Bellare M, Boldyreva A, O'Neill A. Deterministic and efficiently searchable encryption. In Advances in Cryptology-CRYPTO 2007: 27th Annual International Cryptology Conference, Santa Barbara, CA, USA, August 19-23, 2007. Proceedings 27. Berlin Heidelberg: Springer; 2007; 535–552.
4. Goldwasser S, *et al*. Probabilistic encryption. J Comput Syst Sci. 1984; 28(2): 270–299.

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5. Gentry C. A fully homomorphic encryption scheme. PhD thesis. California: Stanford University; 2009.
6. Song DX, *et al*. Practical techniques for searches on encrypted data. In *Proceeding 2000 IEEE Symposium on Security and Privacy (S&P 2000*). 2000; 44–55.
7. Curtmola R, *et al.* Searchable symmetric encryption: Improved definitions and efficient constructions. Journal of Computer Security (JCS). 2011; 19(5): 895–934.
8. Popa RA, *et al*. CryptDB: processing queries on an encrypted database. Commun ACM. 2012; 55(9): 103–111.
9. Tu S, *et al*. Processing analytical queries over encrypted data. Proceedings of the VLDB Endowment (PVLDB). 2013; 6(5): 289–300.
10. Bajaj S, *et al.* Correctdb: SQL engine with practical query authentication. Proceedings of the VLDB Endowment (PVLDB). 2013; 6(7): 529–540.
11. Saini M, Susan S. Deep transfer with minority data augmentation for imbalanced breast cancer dataset. Appl Soft Comput. 2020 Dec 1; 97: 106759.
12. Tayal S. Smart City Ranking Based on Big Data. Doctoral dissertation. Delhi: Delhi Technological University 2020.
13. Qadir AM, Varol N. A review paper on cryptography. In 2019 IEEE 7th international symposium on digital forensics and security (ISDFS). 2019 Jun 10; 1–6.
14. Chachapara K, Bhadlawala S. Secure sharing with cryptography in cloud computing. In 2013 IEEE Nirma University International Conference on Engineering (NUiCONE). 2013 Nov 28; 1–3.
15. Orman H. Creating an Open Community of Cryptographers. In Democratizing Cryptography: The Work of Whitfield Diffie and Martin Hellman. ACM; 2022 Aug 24; 185–212.
16. Gennaro R, Karger P, Matyas S, Peyravian M, Roginsky A, Safford D, Willett M, Zunic N. Two-phase cryptographic key recovery system. Comput Secur. 1997 Jan 1; 16(6): 481–506.
17. Yoshizawa T, Singelée D, Muehlberg JT, Delbruel S, Taherkordi A, Hughes D, Preneel B. A survey of security and privacy issues in v2x communication systems. ACM Comput Surv. 2023 Jan 13; 55(9): 1–36.
18. Egorov M, Wilkison M. ZeroDB white paper. arXiv:1602.07168 [cs.CR]. 2016. CoRR, vol. abs/1602.07168, 2016.
19. Tetali SD, Lesani M, Majumdar R, Millstein T. MrCrypt: Static analysis for secure cloud computations. InProceedings of the 2013 ACM SIGPLAN international conference on Object oriented programming systems languages & applications 2013 Oct 29 (pp. 271-286).
20. AWS. Amazon Aurora. [Online]. available at: https://aws.amazon.com/rds/aurora/.
21. MariaDB. [Online]. Available at: https://mariadb.com/.
22. Bajaj S, *et al.* TrustedDB: A trusted hardware-based database with privacy and data confidentiality. IEEE *Trans Knowl Data Eng* (TKDE). 2014 Mar; 26(3): 752–765.
23. Arasu A, *et al*. Orthogonal security with cipherbase. In 6th Biennial Conference on Innovative Data Systems Research (CIDR). 2013.
24. Sion R. Secure data outsourcing. In 33rd International Conference on Very Large Data Bases (VLDB). 2007; 1431–1432.
25. Arasu A, *et al*. Querying encrypted data. In *2013 IEEE 29th International Conference on Data Engineering (*ICDE). 2013; 1262–1263.
26. Arasu A, *et al*. Querying encrypted data. In Proceedings of the 2014 ACM SIGMOD International Conference on Management of Data. 2014; 1259–1261.
27. Costan V, *et al.* Intel SGX explained. IACR ePrint Archive. 2016.
28. Agrawal D, *et al.* Secure and privacy-preserving database services in the cloud. In *2013 IEEE 29th International Conference on Data Engineering (*ICDE). 2013; 1268–1271.
29. Sahin C, *et al*. Data security and privacy for outsourced data in the cloud. In *2018 IEEE 34th International Conference on Data Engineering (*ICDE). 2018; 1731–1734.
30. SHAttered. [Online]. Shattered.io. 2017. Available from: https://shattered.io/
31. TinyURL. (2024). URL Shortener, Branded Short Links & Analytics. [Online]. Tinyurl. Available from: https://tinyurl.com/app

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32. Shamir A. How to share a secret. Commun ACM. 1979; 22(11): 612–613.
33. Beimel A. Secret-sharing schemes: A survey. In International Conference on Coding and Cryptology (IWCC). 2011; 11–46.
34. Gilboa N, Ishai Y. Distributed point functions and their applications. In EUROCRYPT. 2014; 640–658.
35. Boyle E, Gilboa N, Ishai Y. Function secret sharing,. In EUROCRYPT. 2015; 337–367.
36. Dolev S, *et al.* Accumulating automata and cascaded equations automata for communication-less information theoretically secure multiparty computation. Theor Comput Sci (TCS). 2019; 795: 81–99.
37. Dolev S, *et al*. Privacy-preserving secret shared computations using MapReduce. IEEE *Trans Depend Secur Comput* (TDSC). 2021; 18(4): 1645–1666.
38. Gupta P, *et al.* Obscure: Information-theoretic oblivious and verifiable aggregation queries. Proceedings of the VLDB (PVLDB). 2019; 12(9): 1030–1043.
39. Bater J, *et al*. SMCQL: secure query processing for private data networks. Proceedings of the VLDB (PVLDB). 2017; 10(6): 673–684.
40. Volgushev N, *et al.* Conclave: secure multi-party computation on big data. In EuroSys. 2019; 3:1–3:18.
41. Wang F, *et al*. Splinter: Practical private queries on public data. In NSDI. 2017; 299–313.
42. Stealth Pulsar. [Online]. Available at: http://www.stealthsoftwareinc.com/.
43. Archer DW, *et al.* From keys to databases - real-world applications of secure multi-party computation. IACR Cryptology ePrint. 2018.
44. Emekci F, *et al.* Privacy preserving query processing using third parties. In Proceedings of the 22nd International Conference on Data Engineering (ICDE). 2006; 2
45. Emekci F, *et al*. Dividing secrets to secure data outsourcing. Inf Sci. 2014; 263: 198–210.
46. Xiang T, *et al*. Processing secure, verifiable and efficient SQL over outsourced database. Inf Sci. 2016; 348: 163–178.
47. Zheng W, *et al.* Opaque: An oblivious and encrypted distributed analytics platform. In NSDI. 2017; 283–298.
48. Priebe C, *et al*. Enclavedb: A secure database using SGX. In IEEE *Symposium on Security and Privacy (*SP). 2018; 264–278.
49. Vinayagamurthy D, *et al*. StealthDB: a scalable encrypted database with full SQL query support. PoPETs. 2019; 370–388.
50. Dinh TTA, *et al.* M2R: enabling stronger privacy in mapreduce computation. In USENIX. 2015; 447–462.
51. Schuster F, *et al.* VC3: trustworthy data analytics in the cloud using SGX. In IEEE *Symposium on Security and Privacy* (SP). 2015; 38–54.
52. Brasser F, *et al*. Software grand exposure: SGX cache attacks are practical. In WOOT Proceedings of the 11th USENIX Conference on Offensive Technologies. 2017; 1–11.
53. Gotzfried J, *et al*. Cache attacks on Intel SGX. In EuroSec, Proceedings of the 10th European Workshop on Systems Security. 2017; 2(6p).
54. Moghimi A, *et al*. Cachezoom: How SGX amplifies the power of cache attacks. In Cryptographic Hardware and Embedded Systems (CHES). 2017; 69–90.
55. Lee S, *et al.* Inferring fine-grained control flow inside SGX enclaves with branch shadowing. In USENIX Security. 2017; 557–574.
56. Hahnel¨ M, *et al.* High-resolution side channels for untrusted operating systems. In USENIX ATC. 2017; 299–312.
57. Schwarz M, *et al*. Malware guard extension: Using sgx to conceal cache attacks. In Detection of Intrusions and Malware, and Vulnerability Assessment (DIMVA). 2017; 3–24.
58. Chen G, *et al*. SgxPectre attacks: Stealing intel secrets from SGX enclaves via speculative execution. arXiv preprint. 2018.
59. Costan V, *et al*. Sanctum: Minimal hardware extensions for strong software isolation. In USENIX. 2016; 857–874.

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Implementation of Common Soft Fixed Point with Intuitionistic Soft Fuzzy Metric Space

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Abstract: In this article, we define compatible mappings of type (K) in intuitionistic fuzzy metric space and prove a general Soft fixed point theorem with an application to self mappings on a full intuitionistic fuzzy metric space. Our conclusion broadens and enhances earlier, related findings in the literature.
Keywords: Intuitionistic fuzzy metric space, weakly compatible, compatible mappings of type (K),Soft set,soft fixed point

Introduction

Functional analysis must include the examination of soft fixed points. Studying frequently occurring soft fixed points of mappings that satisfy contractive type constraints has proven to be a very fruitful field of research. The idea of a fuzzy set was created in 1965 by L.A. Zade[11]. Following that, fuzzy metric spaces were introduced by Kramosil and Michalek [13]. George and Veeramani [1] used continuous t-norms to create the idea of fuzzy metric spaces. In 1986, G. Jungck [5] was the first to put forth the concept of compatible mappings. By G. Jungck, P. P. Murthy, and Y. J. Cho in 1993 [6,]. Compatible mappings of type (P) were introduced by M. Kang

[7] and contrasted with compatible mappings of type (A) and compatible mappings. In 1998,. The idea of compatible mappings of type (K) in metric space was recently proposed by Jha et al. [10], who also demonstrated that each compatible mapping of type (K) is independent of the others. A common soft fixed point theorem for compatible mappings of type (K) in intuitionistic fuzzy metric spaces is established in this study with an accompanying illustration. Molodtsov [18] introduced the idea of soft sets as a novel mathematical approach for problem-solving. A parameterized family of subsets of the full set is what Molodtsov[1] refers to as a soft set. Each element of the soft set is thought of as a collection of nearby items, and the soft set is a parameterized family of subsets of the full set. Numerous scholars have been studying the foundations of soft set theory during the past few years. A OR was presented by Maji et al. [19]. They conducted study and discussed the fundamental characteristics of their methods. Pei and Miao [20] examined the connection between information systems and soft sets and redefined the terms intersection and subset of soft sets. [21] with a focus on their fundamental features. Soft matrix theory was introduced by Cagman and Enginoglu [22]

Preliminaries

Definition 2.1. [12] Any non empty set Y and B is a fuzzy set in Y is a mapping with domain Y and [0,1].

Definition 2.2. [1] A binary operation $\diamond : [0, 1] \times [0, 1] \to [0, 1]$ is a continuous t-norm if $\diamond$ is **satisfying the following conditions:**

1. $\diamond$ is commutative and associative;
2. $\diamond$ is continuous;
3. $p \diamond 1 = p$ for all $a \in [0, 1]$;
4. $p \diamond q \leq r \diamond s$ whenever $p \leq r$ and $q \leq s$, and $p, q, r, s \in [0, 1]$.

Definition 2.3. [2] A binary operation $* :[0, 1] \times [0, 1] \to [0, 1]$ is a continuous t-conorm if it satisfies the following conditions:

1. is commutative and associative;
2. *is continuous;
3. $p *0 = p$ for all $p\in [0, 1]$;
4. $p * q \leq r * s$ whenever $p \leq r$ and $q \leq s$, for each $p, q, r, s \in [0, 1]$.

Definition 2.4. [3] Let (Y,P , Q, *, $\diamond$) be an intuitionistic fuzzy metric space.

1. A sequence {xn} in Y is called cauchy sequence if for each $t > 0$ and $P > 0$, $\lim \omega\to\infty P (y_{\omega+p},, t) = 1$and $\lim \omega\to\infty Q (y_{\omega+p},y_{\omega}, t) = 0$.
2. A sequence {yn} in Y is convergent to $y \in Y$ if $\lim \omega\to\infty Q (y_{\omega+p},, t) = 1$ and $\lim \omega\to\infty Q (y_{\omega+p},y_{\omega}, t) = 0$.
3. for each $t > 0$.
4. An intuitionistic fuzzy metric space is said to be complete if every Cauchy sequence is convergent.

Definition 2.5.[10] The self mappings B and R of a fuzzy metric space (Y, P, $*$) are said to be compatible of type (E) iff $\lim\omega \to\infty P(BBy_{\omega}, BRy_{\omega}, t) = 1$, $\lim\omega \to\infty P(BBy_{\omega}, Ry, t) = 1$,
$lim_{\omega\to\infty}$ $P(BRy_{\omega}, Ry, t) = 1$ and $lim_{\omega\to\infty}$ $P(RRy_{\omega}, RBy_{\omega}, t) = 1$,$lim_{\omega\to\infty} P(RRy_{\omega}, By, t) = 1$,
$lim_{\omega\to\infty} P(RRy_{\omega}, By, t) = 1$.

Definition 2.6.[10] The self mappings B and R of a metric space (Y, d) are said to be compatibleof type (K) iff $lim_{\omega\to\infty}$ BBy_ω = Ry and $lim_{\omega\to\infty}$RRy_ω = By, whenever {$y\omega$} is a sequence in Y such that $lim_{\omega\to\infty}$ By_ω = $lim_{\omega\to\infty}$Ry_ω = y for some y in Y.

Definition 2.7 The self mappings B and R of a intuitionistic fuzzy metric space (Y, P, Q, *, ⋄) are said to be compatible of type (K) iff $lim_{\omega\to\infty}$ P(BBy_ω, Ry, t) = 1 and $lim_{\omega\to\infty}$ P(RRy_ω, By, t) = 1, whenever {$y\omega$} is a sequence in Y such that $lim_{\omega\to\infty}$ By_ω =$lim_{\omega\to\infty}$ Ry_ω = y for some x in Y and t
> 0.

Definition 2.10:[18]Let(Y,d) is a metric space and the set (F, B) is a soft set of Y is called null soft set and it is denoted by φ.iffor all ε ∈ B , F(ε) = ϕ. (empty set)

Definition 2.11:[19]Let (Y,d) is a metric space and suppose that the set (F, B) is a soft set over (Y, d) is called absolute soft set. If for all ε ∈ B, F(ε) = Y.

Definition 2.12:**[20]**Let (Y,d) is a metric space and if the difference (F,A) of two soft sets (F,A) and (F,A) over Y is denoted by (F,A)/(F,A)and is defined as
F(e) = H(e)\G(e) for all e ∈ A.

Lemma 2.14. [16] Let (Y, P, Q, *,⋄) be an intuitionistic fuzzy metric space. If there exists a constant k ∈ (0, 1) such that, P($y\omega$+2, $y\omega$+1 kt) ≥ P($y\omega$+1, $y\omega$, t), Q ($y\omega$+1, $y\omega$ + 1,kt) ≤ Q($y\omega$+1, $y\omega$
, t) for every t > 0 and n =1, 2,... then {$y\omega$} is a Cauchy sequence in Y .

Definition 2.13:[21]For any two soft real numbers
1. $\tilde{p} \leq \tilde{q}$, if $\tilde{p}$ (e) ≤ $\tilde{q}$(e), for all e ∈ A.
2. $\tilde{p} \geq \tilde{q}$ if $\tilde{p}$ (e) ≥ $\tilde{q}$ (e), for all e ∈ A.
3. $\tilde{p} < \tilde{q}$ if $\tilde{p}$ (e) < $\tilde{q}$ (e), for all e ∈ A.
4. $\tilde{p} > \tilde{q}$, if $\tilde{p}$ (e) > $\tilde{q}$ (e), for all e ∈ A.

Definition 2.14:[22]The complement of a soft set (P,B) is denoted by (P, B)^c and is defined by
(P, B)^c= (P^c, B) whereF : B→ Q(Y) is mapping given by (β) = Y− P(β) ,∀ β∈ B.

Definition 2.15:[23] Let R be the set of real numbers and C(R)be the collection of all nonempty Bounded subsets of R and C taken as a set of parameters. Then a mapping
A:H→B(R) is said to be a soft real set. It is denoted by (A,H).If specifically (A,H) isa singleton soft set, then identifying (A,H) with the corresponding soft element, it will be said to a soft real number and denoted ã, betc.
Let 0,1 are any two soft real numbers where 0(e) = 0,1 (e) = 1 for all e ∈ H, respectively

Main Results

If the following requirements are met and B, K, L, and T are self mapping on a full intuitionistic soft fuzzy metric space (Y, P, Q, *,),
(i) C (Y) ⊆ T (Y, K(Y) ⊆ L(Y).
There exists k ∈ (0, 1) such that for every $\tilde{x}$, $\tilde{y}$ ∈ $\tilde{Y}$ and t > 0,
P (C $\tilde{x}$, K$\tilde{y}$, kt) ≥ min {P (L$\tilde{x}$, T$\tilde{y}$, t), P (C$\tilde{x}$, L$\tilde{x}$, t), P (K$\tilde{y}$, T$\tilde{y}$, t), P (C$\tilde{x}$, T$\tilde{y}$, t), P (C$\tilde{x}$,K$\tilde{y}$, t), P (L$\tilde{x}$, K$\tilde{y}$, t)},
and
Q (C $\tilde{x}$, K$\tilde{y}$, kt) ≤ max {Q (L$\tilde{x}$, T$\tilde{y}$, t), Q (C$\tilde{x}$, L$\tilde{x}$, t), Q (K$\tilde{y}$, T$\tilde{y}$, t), Q (C$\tilde{x}$, T$\tilde{y}$, t),Q (C$\tilde{x}$, K$\tilde{y}$, t), Q (L$\tilde{x}$, K$\tilde{y}$, t)}.
K and T weakly compatible mappings.
Then C, K, L, and T all have a single common soft fixed point in Y if the pair of mappings (C, L) is compatible of type (K) and one of the mappings is continuous.

Theorem 3.1: If the following requirements are met and B, K, L, and T are self mapping on a full intuitionistic soft fuzzy metric space (Y, P, Q, *,),
(i) C (Y) ⊆ T (Y), K(Y) ⊆ L(Y).
There exists k ∈ (0, 1) such that for every $\tilde{y}$, $\tilde{z}$ ∈ Y and t > 0,
P ($C_{\tilde{y}}$, $K_{\tilde{z}}$, kt) ≥ min{ P($L_{\tilde{y}}$, $T_{\tilde{z}}$,t), P($C_{\tilde{y}}$, $L_{\tilde{z}}$,t), P($K_{\tilde{z}}$, $T_{\tilde{z}}$,t), ($\frac{P(C_{\tilde{y}}, T_{\tilde{z}}, t)}{1+P(C_{\tilde{y}}, T_{\tilde{z}}, t)}$),

P ($C_{\tilde{y}}$
, $K_{\tilde{z}}$
, kt), $\left(\frac{P(L\tilde{y}\ ,K\tilde{z}\ ,t)}{1+P(L\tilde{y}\ ,K\tilde{z}\ ,t)}\right)$ } and

Q ($C_{\tilde{y}}$, $K_{\tilde{z}}$, kt) ≤ max{Q (L_y , $T_{\tilde{z}}$,t), Q (G_y , $L_{\tilde{z}}$,t), Q ($K_{\tilde{z}}$, $T_{\tilde{z}}$,t),
$\left(\frac{Q(C\tilde{y}\ ,T\tilde{z}\ ,t)}{1+Q(C\tilde{y}\ ,T\tilde{z}\ ,t)}\right)$, Q (C , K , kt), $\left(\frac{Q(L\tilde{y}\ ,K\tilde{z}\ ,t)}{1+Q(L\tilde{y}\ ,K\tilde{z}\ ,t)}\right)$

$\tilde{y}$ $\tilde{z}$

K and T weakly compatible mappings.
Then C, K, L, and T all have a single common soft fixed point in $\tilde{Y}$ if the pair of mappings (C, L) is compatible of type (K) and one of the mappings is continuous

Proof: Suppose $\tilde{y}_0$ be any random point in $\tilde{Y}$,constract a $\{\tilde{z}_n\}$ in $\tilde{Y}$
as a means to $\tilde{z}_{2\omega-1} = T\tilde{y}_{2\omega-1}$

= $C\tilde{y}_{2\omega-2}$ & $\tilde{z}_{2\omega} = L\,\tilde{y}_{2\omega} = K\,\tilde{y}_{2\omega+1}$, ω = 1,2,3,4 this can be done by (1),
By using contractive condition ,we obtain
P ($\tilde{z}_{2\omega+1}$, $\tilde{z}_{2\omega+2}$, kt) = P (C $\tilde{y}_{2\omega}$, $K\tilde{y}_{2\omega+1}$, kt)
P ($\tilde{z}_{2\omega+1}$, $\tilde{z}_{2\omega+2}$, kt) ≥ min{P($L\tilde{y}_{2\omega}$,T $\tilde{y}_{2\omega+1}$, t), P($C\tilde{y}_{2\omega}$, $L\tilde{y}_{2\omega+1}$, t), P($K\tilde{y}_{2\omega+1}$, T $\tilde{y}_{2\omega+1}$, t), P($L\tilde{y}_{2\omega}$,
$K\tilde{y}$
$2\omega+1$
, t), $\frac{P(L\tilde{y}2\omega\ ,K\tilde{y}2\omega+1\ ,t)}{1+P(L\tilde{y}2\omega\ ,\tilde{K}y2\omega+1\ ,t)}$,
$\frac{P\ (C\tilde{y}2\omega,\tilde{T}y2\omega+1\ ,t)}{1+P\ (\tilde{C}y2\omega,\tilde{T}y2\omega+1\ ,t)}$ }
≥ min{P($\tilde{z}_{2\omega}$, $\tilde{z}_{2\omega+1}$, t), P($\tilde{z}_{2\omega+1}$, $\tilde{z}_{2\omega+1}$, t), P ($\tilde{z}_{2\omega+1}$, $z_{2\omega+1}$, t), P($\tilde{z}_{2\omega}$, $\tilde{z}_{2\omega+1}$, t),

$\frac{P(z2\omega+1\ ,\tilde{z}2\omega+1\ ,t)}{1+P(z2\omega+1\ ,\tilde{z}2\omega+1\ ,t)}$,
$\frac{P\ (\tilde{z}2\omega\ ,\tilde{z}2\omega,t)}{1+P\ (\tilde{z}2\omega,z2\omega\ ,t)}$ }

P ($\tilde{z}_{2\omega+1}$, $\tilde{z}_{2\omega+2}$, kt) ≥ min{P($\tilde{z}_{2\omega}$, $\tilde{z}_{2\omega+1}$, t), P ($\tilde{z}_{2\omega+1}$, $\tilde{z}_{2\omega+1}$, t), P ($\tilde{z}_{2\omega+1}$, $z_{2\omega+1}$, t), P($\tilde{z}_{2\omega}$, $\tilde{z}_{2\omega+1}$, t),
$\frac{P(z2\omega+1\ ,\tilde{z}2\omega+1\ ,t)}{1+P\ (\tilde{z}2\omega\ ,\tilde{z}2\omega,t)}$ } P(z2ω+1 ,$\tilde{z}$2ω+1 ,t)
P ($\tilde{z}$2ω,z2ω ,t)
P ($\tilde{z}_{2\omega+1}$, $\tilde{z}_{2\omega+2}$, kt) ≥ min{P($\tilde{z}_{2\omega}$, $\tilde{z}_{2\omega+1}$, t), 1, 1, P($\tilde{z}_{2\omega}$, $\tilde{z}_{2\omega+1}$, t),1,1 }

P ($\tilde{z}_{2\omega+1}$, $\tilde{z}_{2\omega+2}$, kt) ≥ P($\tilde{z}_{2\omega}$, $\tilde{z}_{2\omega+1}$, t)
Similarly, we have P($\tilde{z}_{2\omega}$, $\tilde{z}_{2\omega+1}$, t) ≥ P($\tilde{z}_{2\omega-1}$, $\tilde{z}_{2\omega}$, t) (1) So we get P($\tilde{z}_{2\omega+2}$, $\tilde{z}_{2\omega+1}$, t) ≥ P($\tilde{z}_{2\omega+1}$, $\tilde{z}_{2\omega}$, t)
Also ,we get
Q($\tilde{z}_{2\omega+1}$, $\tilde{z}_{2\omega+2}$, kt) = Q($C\tilde{z}_{2\omega}$, $K\tilde{z}_{2\omega+1}$,kt)
Q($\tilde{z}_{2\omega+1}$, $\tilde{z}_{2\omega+2}$,kt) ≤ max{Q($L\tilde{y}_{2\omega}$,T $\tilde{y}_{2\omega+1}$, t), Q ($C\tilde{y}_{2\omega}$, $L\tilde{y}_{2\omega+1}$, t), Q ($K\tilde{y}_{2\omega+1}$, T $\tilde{y}_{2\omega+1}$, t),

Q($L\tilde{y}_{2\omega}$
, $K\tilde{y}$
$2\omega+1$
, t), $\frac{Q(L\tilde{y}2\omega\ ,\tilde{K}y2\omega+1\ ,t)}{1+Q(L\tilde{y}2\omega\ ,\tilde{K}y2\omega+1\ ,t)}$,
$\frac{Q\ (C\tilde{y}2\omega,T\tilde{y}2\omega+1\ ,t)}{1+Q\ (C\tilde{y}2\omega,\tilde{T}y2\omega+1\ ,t)}$ }

$\leq \max \{Q(\tilde{z}_{2\omega}, \tilde{z}_{2\omega+1}, t), Q(\tilde{z}_{2\omega+1}, \tilde{z}_{2\omega+1}, t), Q(\tilde{z}_{2\omega+1}, z_{2\omega+1}, t), Q(\tilde{z}_{2\omega}, \tilde{z}_{2\omega+1}, t),$

$$\frac{Q(z2\omega+1, \tilde{z}2\omega+1, t)}{1+Q(\tilde{z}2\omega, z2\omega, t)} Q(z2\omega+1, \tilde{z}2\omega+1, t), \frac{Q(\tilde{z}2\omega, \tilde{z}2\omega, t)}{1+Q(\tilde{z}2\omega, z2\omega, t)} \}$$

$Q(\tilde{z}_{2\omega+1}, \tilde{z}_{2\omega+2}, kt) \leq \max \{Q(z_{2\omega}, \tilde{z}_{2\omega+1}, t), Q(\tilde{z}_{2\omega+1}, z_{2\omega+1}, t), Q(\tilde{z}_{2\omega+1}, z_{2\omega+1}, t),$
$Q(\tilde{z}_{2\omega}, z_{2\omega+1}, t), \frac{Q(z2\omega+1, \tilde{z}2\omega+1, t)}{1+Q(\tilde{z}2\omega, z2\omega, t)} Q(z2\omega+1, \tilde{z}2\omega+1, t), \frac{Q(\tilde{z}2\omega, \tilde{z}2\omega, t)}{1+Q(\tilde{z}2\omega, z2\omega, t)} \}$

$Q(\tilde{z}_{2\omega+1}, z_{2\omega+2}, kt) \leq \{Q(z_{2\omega}, \tilde{z}_{2\omega+1}, t), 0, 0, Q(z_{2\omega}, z_{2\omega+1}, t), 0, 0\}$ $Q(\tilde{z}_{2\omega+1}, z_{2\omega+2}, kt) \leq Q(z_{2\omega}, \tilde{z}_{2\omega+1}, t)$

Similarly we have

$Q(\tilde{z}_{2\omega}, \tilde{z}_{2\omega+1}, t) \leq Q(\tilde{z}_{2\omega-1}, \tilde{z}_{2\omega}, t)$ (2) So we get $Q(\tilde{z}_{2\omega+2}, \tilde{z}_{2\omega+1}, t) \leq Q(\tilde{z}_{2\omega+1}, \tilde{z}_{2\omega}, t)$

From (1),(2) and (2.13), We comprehend that $\{z\}$ is a Cauchy sequence in Y .

But $(\tilde{Y}, \tilde{P}, \tilde{Q}, *, \diamond)$ is complete,So there are a soft point $\tilde{z}$ in $\tilde{Y}$ .Such that $\{\tilde{z}_{\omega}\} \rightarrow \tilde{q}$. Also we have $\{C\tilde{y}_{2\omega-2}\}, \{T\tilde{y}_{2\omega-1}\}, \{L\tilde{y}_{2\omega}\}, \{K\tilde{y}_{2\omega-1}\} \rightarrow \tilde{q}$.

Since (C, L) is compatible of type (K) and one of the mapping is continuous. So using (2.15),We obtain $C\tilde{q} = L\tilde{q}$. (3)

Since $C\tilde{(Y)} \subseteq T\tilde{(Y)}$,Then one can find a point $\tilde{v}$. in $\tilde{Y}$, as a means to $C\tilde{q} = T\tilde{v}$.

Now by contractive condition, we obtain

$$P(C\tilde{q}, K\tilde{v}, kt) \geq \min \{P(L\tilde{q}, T\tilde{v}, t), P(C\tilde{q}, L\tilde{v}, t), P(K\tilde{v}, T\tilde{v}, t), \frac{P(L\tilde{q}, K\tilde{v}, t)}{1+(L\tilde{q}, v, t)}, P(C\tilde{q}, K\tilde{v}, t), \frac{P(C\tilde{q}, T\tilde{v}, t)}{1+(C\tilde{q}, \tilde{v}, t} \}$$

$$= \min \{P(C\tilde{q}, C\tilde{q}, t), P(C\tilde{q}, C\tilde{q}, t), P(K\tilde{v}, C\tilde{q}, t), \frac{P(C\tilde{q}, K\tilde{v}, t)}{1+(C\tilde{q}, v, t)}, P(C\tilde{q}, C\tilde{q}, t), \frac{P(C\tilde{q}, K\tilde{v}, t)}{1+(C\tilde{q}, \tilde{v}, t} \}$$

$$= \min \{P(C\tilde{q}, C\tilde{q}, t), P(C\tilde{q}, C\tilde{q}, t), P(K\tilde{v}, C\tilde{q}, t), P(K\tilde{v}, C\tilde{q}, t), P(C\tilde{q}, C\tilde{q}, t), P(K\tilde{v}, C\tilde{q}, t)\}$$

$P(C\tilde{q}, K\tilde{v}, kt) \geq P(C\tilde{q}, K\tilde{v}, t)$ (4)

Also ,we obtain

$$Q(C\tilde{q}, K\tilde{v}, kt) \leq \max \{Q(L\tilde{q}, T\tilde{v}, t), Q(C\tilde{q}, L\tilde{v}, t), Q(K\tilde{v}, T\tilde{v}, t), \frac{Q(L\tilde{q}, K\tilde{v}, t)}{1+Q(L\tilde{q}, \tilde{K}v, t)}, Q(C\tilde{q}, K\tilde{v}, t), \frac{Q(C\tilde{q}, T\tilde{v}, t)}{1+Q(C\tilde{q}, T\tilde{v}, t} \}$$

$$= \max \{Q(C\tilde{q}, C\tilde{q}, t), Q(C\tilde{q}, C\tilde{q}, t), Q(K\tilde{v}, C\tilde{q}, t), \frac{Q(C\tilde{q}, K\tilde{v}, t)}{1+Q(C\tilde{q}, K\tilde{v}, t)}, Q(C\tilde{q}, C\tilde{q}, t), \frac{Q(C\tilde{q}, \tilde{K}v, t)}{1+Q(C\tilde{q}, K\tilde{v}, t} \}$$

= max { Q(Cq̃,Cq̃,t),Q(Cq̃,Cq̃,t),Q(Kṽ,Cq̃,t), Q(Kṽ,Cq̃,t) ,Q(Cq̃,Cq̃,t), Q(Kṽ,Cq̃,t) }

Q(Cq̃,Kṽ,kt) ≤ Q(Cq̃,Kṽ,t) (5)

Now from (4) and (5) and Lemma (2.14), We obtain Cq̃ = Kṽ
Thus we obtain Cq̃ = Lq̃ =Kq̃ =Tq̃ (6)
To prove Rq̃ = q̃
P(Cq̃, $Kz_{2\omega+1}$ kt) ≥min {P(Lq̃, $Tz_{2\omega+1}$, t) , P(Cq̃, $Lz_{2\omega+1}$, t) ,P($Kz_{2\omega+1}$, $Tz_{2\omega+1}$, t),

$\frac{P(Lq̃,Tz_{2\omega+1},t)}{1+P(Lq,Tz_{2\omega+1},t)}$
, P(Cq̃, Kz
$_{2\omega+1}$
, t) , $\frac{P(Cq̃,Tz_{2\omega+1},t)}{1+P(Cq̃,Tz_{2\omega+1},t)}$ }

let as n → ∞, we obtain

P(Cq̃, q̃,kt) ≥ min {P(Lq̃, q̃,t),P (Cq, q̃,t),P(q̃, q̃,t), $\frac{P(Lq̃,q̃,t)}{1+P(Lq̃,q̃,t)}$

, P(Cq̃, q̃,t), $\frac{P(Cq̃,q̃,t)}{1+P(Cq̃,q̃,t)}$ }
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P(Cq̃, q̃,kt) = min {P(Cq̃, q̃,t),P (Cq̃, q̃,t),1 ,1 , P(Cq̃, q̃,t), 1 }

P(Cq̃, q̃,kt) ≥ P (Cq̃, q̃,t) (7)

Q(Cq̃, $Kz_{2\omega+1}$ kt) ≤ max {Q(Lq̃, $Tz_{2\omega+1}$, t) , Q(Cq̃, $Lz_{2\omega+1}$, t) ,Q($Kz_{2\omega+1}$, $Tz_{2\omega+1}$, t),

$\frac{Q(Lq̃,Tz_{2\omega+1},t)}{1+Q(Lq̃,Tz_{2\omega+1},t)}$,Q(Cq̃, Kz

let as n → ∞, we obtain
$_{2\omega+1}$
, t) , $\frac{Q(Cq̃,Tz_{2\omega+1},t)}{1+Q(Cq̃,Tz_{2\omega+1},t)}$ }

Q(Cq̃, q̃,kt) ≤max {Q(Lq̃, q̃,t), Q(Cq̃, q̃,t), Q(q̃, q̃,t), $\frac{Q(Lq̃,q̃,t)}{1+Q(Lq̃,q̃,t)}$
, Q(Cq̃, q̃,t), $\frac{Q(Cq̃,q̃,t)}{1+Q(Cq̃,q̃,t)}$ }

$Q(C\tilde{q}, \tilde{q},kt) = \max \{Q(L\tilde{q}, \tilde{q},t), Q(C\tilde{q}, \tilde{q},t), 0, \frac{Q(C\tilde{q},\tilde{q},t)}{1+Q(C\tilde{q},\tilde{q},t)}, Q(C\tilde{q}, \tilde{q},t), \frac{Q(C\tilde{q},\tilde{q},t)}{1+Q(C\tilde{q},\tilde{q},t)}\}$

$Q(C\tilde{q}, \tilde{q}, kt) = \max\{Q(C\tilde{q}, \tilde{q},t), Q(C\tilde{q}, \tilde{q},t), 0, \frac{Q(C\tilde{q},\tilde{q},t)}{1+Q(C\tilde{q},\tilde{q},t)}, Q(C\tilde{q}, \tilde{q},t), \frac{Q(C\tilde{q},\tilde{q},t)}{1+Q(C\tilde{q},\tilde{q},t)}\}$

$Q(C\tilde{q}, \tilde{q}, kt) = \max \{Q(C\tilde{q}, \tilde{q},t), Q(C\tilde{q}, \tilde{q},t), 0, 0, Q(C\tilde{q}, \tilde{q},t), 0\}$

$Q(C\tilde{q}, \tilde{q},kt) \leq \max Q(C\tilde{q}, \tilde{q},t)$ (8)

Now from (7) and (8) & lemma 2.14, we obtain

$C\tilde{q} = \tilde{q}$

That being the case ,we obtain $C\tilde{q} = L\tilde{q} = \tilde{q}$ So $\tilde{q}$ is a comman soft point of C and L. (9)

As well we obtain $K\tilde{v} = T\tilde{v} = \tilde{q}$ (10)

Since K and T are weakly compatible ,we have $T K\tilde{v} = K T\tilde{v}$.

So from (6),we obtain $T\tilde{q} = K\tilde{q}$ Again ,we obtain (11)

$P(C\tilde{z}_{2\omega-2}, K\tilde{q}, kt) \geq \min\{P(L\tilde{z}_{2\omega-2}, T\tilde{q},t), P(C\tilde{z}_{2\omega-2}, L\tilde{q}, t), P(K\tilde{q},T\tilde{q},t), \frac{P(C\tilde{z}_{2\omega-2}, T\tilde{q},t)}{1+P(C\tilde{z}_{2\omega-2}, T\tilde{q},t)},$

$P(C\tilde{z}_{2\omega-2}, L\tilde{q}, t), \frac{P(C\tilde{z}_{2\omega-2}, T\tilde{q},t)}{1+P(C\tilde{z}_{2\omega-2}, T\tilde{q},t)}\}$

Taking limit $n \to \infty$,We have

$P(\tilde{q}, K\tilde{q}, kt) \geq \min\{P(\tilde{q},T\tilde{q},t), P(\tilde{q}, L\tilde{q}, t), P(K\tilde{q},T\tilde{q},t), \frac{P(\tilde{q}, T\tilde{q},t)}{1+P(\tilde{q}, T\tilde{q},t)}\}$

$P(\tilde{q}, K\tilde{q}, kt) \geq \min\{P(\tilde{q},T\tilde{q},t), P(\tilde{q}, T\tilde{q}, t), P(T\tilde{q},T\tilde{q},t), \frac{P(\tilde{q}, T\tilde{q},t)}{1+P(\tilde{q}, T\tilde{q},t)}, P(\tilde{q}, T\tilde{q}, t), \frac{P(\tilde{q}, T\tilde{q},t)}{1+P(\tilde{q}, T\tilde{q},t)}, P(T\tilde{q}, T\tilde{q}, t), \frac{P(\tilde{q}, T\tilde{q},t)}{1+P(\tilde{q}, T\tilde{q},t)}\}$

$P(\tilde{q}, K\tilde{q}, kt) \geq \min\{P(\tilde{q},T\tilde{q},t), P(\tilde{q}, T\tilde{q}, t), 1, P(\tilde{q}, T\tilde{q}, t), 1, P(\tilde{q}, T\tilde{q}, t)\}$

$P(\tilde{q}, K\tilde{q}, kt) \geq P(\tilde{q},T\tilde{q},t)$ (12)

As well ,we obtain

$Q(C\tilde{z}$ $Q(C\tilde{z}$

2ω−2

2ω−2
, Kq̃, kt)≤ max {Q(Lž
, Lq̃, t), Q(Cž2ω−2 ,Tq̃,t) }
1+ Q(Cž2ω−2 ,Tq̃,t)
2ω−2
,Tq̃,t), Q(Cž
2ω−2
, Lq̃, t),Q(Kq̃,Tq̃,t), Q(Cž2ω−2 ,Tq̃,t) ,
1+ Q(Cž2ω−2 ,Tq̃,t)

Taking limit n → ∞ ,We have
Q(q̃ , Kq̃, kt) ≤ max{Q(q̃,Tq̃,t), P(q̃ , Lq̃, t),Q(Kq̃,Tq̃,t), Q(q̃ ,Tq̃,t)
1+ Q(q̃ ,Tq̃,t)

Q(q̃ , Kq̃, kt) ≤ max {Q(q̃,Tq̃,t), Q(q̃ , Tq̃, t),Q(Tq̃,Tq̃,t), Q(q̃ ,Tq̃,t)
1+ Q(q̃ ,Tq̃,t)
, Q(q̃ , Tq̃, t), Q(q̃ ,Tq̃,t) }
1+ Q(q̃ ,Tq̃,t)

, Q(Tqq̃, Tq̃, t), Q(q̃ ,Tq̃,t) }
1+ Q(q̃ ,Tq̃,t)

Q(q̃ , Kq̃, kt) ≤ max {Q(q̃,Tq̃,t), Q(q̃ , Tq̃, t), 1 , Q(q̃, Tq̃ , t), 1, Q(q̃ , Tq̃ , t)}

Q(q̃ , Kq̃ , kt) ≤ Q(q̃,Tq̃,t) (13)
Now from (12) and (13) and Lemma(2.14)
Since we have Tq̃ = Kq̃ = q̃ (14)
That being the case ,we obtain q̃ is a ordinary soft fixed point of C,K,L and T. For Uniqueness
Suppose r̃ be the another ordinary soft fixed point then Cr̃ = Lr̃ = Kr̃ Rr̃ = r̃

P(Cq̃, K̃r, kt) ≥ min {P((Lq̃, Tr̃, t), P(Cq̃, Lr̃, t) , P(K̃r, Tr̃, t), P(Cq̃,Tr̃,t)
1+ P(Cq̃,Tr̃,t)
, P(Cq̃, Kr̃, t) , P(Lq̃,Tr̃,t) }
1+P (Lq̃,Tr̃,t)

P(Cq̃, K̃r, kt) ≥ min {P((Cq̃, Kr̃, t), P(Cq̃, C̃r, t) , P(K̃r, Kr̃, t), P(Cq̃,Kr̃,t)
1+P (Cq̃,Kr̃,t)
, P(Cq̃, Kr̃, t) , P(Cq̃,Kr̃,t) }
1+P (Lq̃,Kr̃,t)
P(Cq̃, Kr̃, kt) = min {P((Cq̃, K̃r, t), P(Cq̃, Cr̃, t) , 1, , P(Cq̃, Kr̃, t) , P(Cq̃, K̃r, t), P(Cq̃, Kr̃, t) }

(Cq̃, Kr̃, kt) ≥ P((Cq̃, K̃r, t) (15)
As well we obtain

Q(Cq̃, Kr̃, kt) ≤ max {Q((Lq̃, T̃r, t), Q(Cq̃, Lr̃, t) , Q(Kr̃, T
, t), Q(Cq̃,Tr̃,t)
1+ Q(Cq̃,Tr̃,t)
, Q(Cq̃, Kr̃, t) , Q(Lq̃,Tr̃,t) }
1+Q (Lq̃,Tr̃,t)

Q(Cq̃, Kr̃, kt) ≤ max {Q((Cq̃, Kr̃, t), Q(Cq̃, C̃r, t) , Q(Kr̃, K̃r, t), Q(Cq̃,Kr̃,t)
1+Q (Cq̃,Kr̃,t)
}

, Q(Cq̃, Kr̃, t) , $\frac{Q(Cq̃,Kr̃,t)}{1+ Q(Lq̃,Kr̃,t)}$

Q(Cq̃, Kr̃, kt) ≤ max{Q((Cq̃, K
, t), Q(Cq̃, Cr̃, t) , 0, , Q(Cq̃, Kr̃, t) , Q(Cq̃, K
, t), Q(Cq̃, Kr̃, t) }

Q(Cq̃, K̃r, kt) ≤ Q((Cq̃, Kr̃, t) (16)
From (15),(16) and Lemma(2.14)
We obtain Cq̃ = Kq̃ ,This implies Cq̃ = Cr̃ ,and q̃ is a unique soft fixed point.

Remarks: Our result is also true for the pair of mappings (A, S) is compatible of type (E) in place of compatible mapping of type (K). Our result extends and generalizes the results of M. Verma and R.S. Chandel [13], Jha *et al.*[11] Manandhar *et al.* and improve the result of Manandhar *et al.* [10]. Also, our result improves other similar results in literature.

References

1. George, P. Veeramani, On some results in fuzzy metric spaces Fuzzy Sets and Systems, **64**
2. (1994), 395-399.
3. Schweizer, Sklar, "A. Statistical metric spaces", Pacific J. of Math., 10 (1960) 314-334.
4. Alaca, Turkoglu.D, Yildiz.C, "Fixed points in intuitionistic fuzzy metric Spaces", Chaos, Solitons and Fractals, **29**(2006), 1073-1078.
5. G. Jungck and B. E. Rhoades, Fixed Point for Set Valued functions without Continuity, Indian J. Pure Appl. Math., 29(3), (1998), 771- 779.
6. G. Jungck, Compatible mappings and common fixed points, Internat. J. Math. Math. Sci.,
7. **9**(1986), pp. 771–779.
8. G. Jungck, P. P. Murthy and Y. J. Cho, Compatible mappings of type (A) and common fixed points, Math. Japonica, **38**(1993), 381–390.
9. H. K. Pathak, Y. J. Cho, S. S. Chang and S. M. Kang, Compatible mappings of type (P) and fixed point theorem in metric spaces and Probabilistic metric spaces, Novi Sad J. Math., Vol. 26(2)(1996), 87-109.
10. J.H. Park, "Intuitionistic fuzzy metric spaces", Chaos, Solitons & Fractals **2**(2004), 1039–1046. [9]. K. B. Manandhar, K. Jha and G. Porru. Common Fixed Point Theorem of Compatible
11. Mappings of Type (K) in Fuzzy Metric Space, Electronic J. Math. Analysis and Appl,
12. **2**(2)(2014), 248-253.
13. [10]. K. B. Manandhar, K. Jha and H. K. Pathak, A Common Fixed Point Theorem for mpatible
    Mappings of Type (E) in Fuzzy Metric space, Appl Math. Sci, **8** (2014), 2007 – 2014.
14. K. Jha, V. Popa and K.B. Manandhar, A common fixed point theorem for compatible mapping of type (K) in metric space, Internat. J. of Math. Sci. & Engg. Appl. (IJMSEA), **8** (I) (2014), 383-391.
15. L.A. Zadeh, Fuzzy sets, Inform and Control **8** (1965), 338-353.
16. M. Verma and R. S. Chandel, Common fixed point theorem for four mappings in intuitionistic fuzzy metric space using absorbing Maps, IJRRAS 10 (2) (2012), 286 – 291.
17. [14]. O. Kramosil and J. Michalek, Fuzzy metric and statistical metric spaces, Kybernetika **11**
18. (1975), 326-334.
19. R. P. Pant, A common fixed point theorem under a new condition, Indian. J. Pure Appl. Math., 30 (2) (1999) 147–152.
20. S. Sharma., Kutukcu,and R.S. Rathore , "Common fixed point for Multivalued mappings in intuitionistic fuzzy metricspace", Communication of Korean Mathematical Society, **22** (3),(2007), 391-399.
21. Y.J. Cho, H.K. Pathak, S.M. Kang, J.S. Jung, Common fixed points of compatible maps of type (β) on fuzzy metric spaces, Fuzzy Sets and Systems **93** (1998), 99-111.
22. Molodtsov, D. **Soft –theory-first result**, Comput.Math.Appl. 37,19-31 (1999).
23. Moji, P.K. Roy, A.R, Bisbas,R.**An application of soft sets in a decision making problem**, Comput. Math. Appl. 44, 1077 – 1083 (2002).
24. Moji, P.K.Roy, A.R,Bisbas,R.**Soft set theory**, Computer. Math. Appl. 45, 555 – 562(2003).
25. Ali, M.I., Feng, F., Liu, X., Min, W.K., Shabir, M.: **On some new operations in soft settheory**. Computer. Math. Appl. 57, 1547–1553 (2009).
26. C¸aˇgman, N., Enginoglu, S.: **Soft set theory uni-int decision making**. Eur. J. Oper. Res. 207, 845–855 (2010).
27. Babitha, K.V., Sunil, J.J.: **Soft set relations and functions**. Comput. Math., Appl. 60, 1840–1849 (2010).

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